Strategic Uncertainty in Financial Markets: Evidence from a Consensus Pricing Service*

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Abstract

This paper studies the ability of a consensus pricing service to reduce uncertainty among dealer banks in the over-the-counter options market. The analysis is based on the structural estimation of a model of learning from prices. The estimation yields two empirical measures of uncertainty: uncertainty about option values and strategic uncertainty about competitors' valuations. The main contribution of the consensus prices is to reduce strategic uncertainty, especially in the most opaque segments of the options market. The results stress the importance of pricing benchmarks for aggregating dispersed information and creating a shared understanding of market conditions in opaque market structures.

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1 Introduction

Prices serve a dual purpose. They aggregate dispersed information about gains from trade. At the same time, they also help market participants to coordinate their actions. Empirical work on the informational value of prices typically focuses on the first aspect. However, the ability of prices to reduce strategic uncertainty, that is "uncertainty concerning the actions and beliefs (and beliefs about the beliefs) of others" adopting a definition by Morris and Shin (2002), can be of equal importance. This is especially true in markets with strong coordination motives. In intermediated markets, for example, intermediaries have to hold a view about the gains from trade that can be achieved between buyers and sellers. But they also have to form beliefs about their peers' behavior as the prices they quote have to factor in their ability to trade and share risks with other intermediaries, for example risk that derives from holding inventory. Publicly observed prices can help to avoid costly coordination failures by establishing reference points for trading and create a common understanding of market conditions among market participants.[1]

A key challenge to assessing the importance of prices for reducing strategic uncertainty is measurement. Strategic uncertainty is a concept that is based on market participants' beliefs and data on beliefs are typically not available.[2] Often research uses empirical proxies to measure strategic uncertainty, such as the cross-sectional dispersion of professional forecasts of interest rates or stock market returns. But this still requires a model that maps these proxies into market participants' beliefs and disciplines how beliefs react to information contained in prices. Forecast dispersion, for example, can reflect disagreement among professional forecasters rather than market participants' uncertainty about each others' actions or beliefs. More direct observations on market participants' assessments of market conditions would be preferable, ideally for markets in which strategic uncertainty is likely to play an important

[1]Episodes of financial market freezes provide ample evidence for coordination failure in the absence of market prices. Lowenstein (2000) (p. 159), for example, gives a vivid account of the bond market at the height of the LTCM crisis on August 31, 1998: "It was as if a bomb had hit; traders looked at their screens, and the screens stared blankly back. [...] So few issues traded, you had to guess where they were."

[2]However, eliciting higher-order beliefs has a long tradition in the experimental literature, especially building on Nagel (1995). More recently, research has started to collect survey data on firm and investor beliefs about financial prices and macroeconomic conditions, for example Coibion et al. (2021) and Giglio et al. (2021).

role. An obvious candidate is financial over-the-counter (OTC) markets. The OTC market structure depends on intermediaries for trading and has limited price transparency. In many financial OTC markets consensus pricing services have sprung up to make up for the lack of publicly available price data. These services collect estimates of asset values from their subscribers, typically the main intermediaries in the market, and in return provide them with a consensus price, an aggregate price calculated from their estimates. The consensus price should reflect the current market value of the asset. As all major intermediaries contribute to the service, the consensus pricing setup constitutes a contained informational environment and is an ideal setting to study the impact of prices on strategic uncertainty. It tracks the beliefs of a well-defined group of highly-sophisticated market participants over time. At the same time, given the opaque OTC market structure, the consensus price is likely the only price that is jointly observed by all members of this group.

In this paper we develop a novel approach to measuring strategic uncertainty based on the structural estimation of a model of learning from prices. We apply this framework to the OTC derivatives market and quantify the importance of consensus price information for reducing strategic uncertainty among market participants. The empirical analysis is based on a new data set of price submissions that large dealer banks, highly sophisticated market participants, make to the main consensus pricing service in OTC options market. We use a structural estimation to obtain empirical measures of uncertainty that are based on dealer banks' model-implied beliefs. We prove identification of the learning model. Observing both a panel of individual dealer banks' price submissions and the aggregate consensus price feedback is crucial for identifying the structural parameters of the model. We also measure how efficient the consensus price is in aggregating dispersed information. In the model, dealer banks learn about a latent and time-varying option value from two types of signals: a noisy private signal and a consensus price. The consensus price is modelled as an endogenous signal: it is the average expectation across dealers of an option's value plus noise. Each dealer bank is uncertain about the current value of the option and other dealers' expectations of this value. We use the variance of a dealer bank's posterior beliefs about option value and competing dealers' average expectations to measure these two dimensions of uncertainty. To gauge the informational value of consensus prices, we perform counterfactual experiments on the option market's information structure. As we model the consensus price to be an endogenous signal, we can counterfactually shut down the consensus pricing services while holding the total amount of information distributed across market participants constant.

We find that dealer banks' strategic uncertainty as well as their uncertainty about option values varies substantially across the different segments of the OTC options market. Some market segments overlap with exchange-based options trading for which price data is publicly available. Here, dealer banks appear to be fairly confident both in their option valuations and in their ability to judge competitors' valuations. Options contracts with extreme contract terms are exclusively OTC traded. For these contracts, we find considerable valuation and strategic uncertainty. Our model implies that dealer-banks' beliefs are normal distributions. This allows us to quantify their uncertainty in terms of posterior intervals. We find that a dealer bank's 95 percent posterior interval around its current best estimate of competitors' option valuations is as wide as 6.5 volatility points for option contracts with the highest strategic uncertainty. This corresponds to roughly half the contract value. Counterfactually eliminating the consensus price feedback for dealer banks increases this strategic uncertainty by up to 37 percent. We also infer from this counterfactual experiment that dealer banks do not rely heavily on the consensus price feedback to reduce their uncertainty about option values. The reduction in valuation uncertainty is at most 4 percent in the most opaque market segment. These results imply that the consensus price information is most important for reducing strategic uncertainty. This impact is strongest for option contracts with extreme contract terms. This reflects the scarcity of publicly available price information in these market segments. It also stresses the importance of publicly observed market data, such as financial benchmarks, for establishing a shared understanding of market conditions in markets with limited price transparency. Finally, to judge the informational efficiency of the consensus price, we compare it to a counterfactual price that perfectly aggregates the private information dispersed across dealer banks. We find that in market segments that overlap with exchange-based trading, the consensus price is almost fully efficient in aggregating information, both information about option values and information about competitors' valuations. However, in the most opaque market segments, a fully efficient information aggregation mechanism could reduce strategic uncertainty by up to an additional 60% and valuation uncertainty by up to 33%.

The estimation framework developed in this paper makes a methodological contribution by showing how to structurally identify the informativeness and informational efficiency of prices. The modern theoretical framework for these questions dates back to the early 1980s, with seminal contributions by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981).[3] Vives (1997) highlights the importance of the mix between public and private information for the speed of information aggregation. However, as pointed out by Townsend (1983), determining the informational content of the price process in a dynamic equilibrium context poses significant technical challenges. Most structural empirical analyses of price formation avoid these problems by assuming that asset values become common knowledge after a finite number of periods, e.g., following Easley et al. (1996)'s information structure. But this assumption prevents long-lasting belief heterogeneity. A time-varying latent fundamental value paired with privately informed market participants is a key source of belief heterogeneity and, hence, strategic uncertainty in our model. To solve the dynamic signal extraction problem and structurally estimate the model, we adopt an iterative algorithm previously used in Nimark (2014) and Barillas and Nimark (2017). We show that observing belief updating dynamics at the level of the individual institution is key for identifying the structural parameters of the model. Modelling the consensus price as an endogenous public signal allows us to conduct counterfactual experiments on the market's information structure to evaluate the strength of informational externalities caused by public information (Amador and Weill (2012)).

There is a large literature that uses the cross-sectional dispersion among professional forecasters to study informational frictions (Coibion and Gorodnichenko (2012), Andrade et al. (2016)). A major aim of this literature is to understand the expectation formation process. The insights gained are then used to discriminate between alternative models, extrapolate to a wider group of market participants than just professional forecasters, and study macroeconomic consequences. In this paper we have a different objective. We want to measure uncertainty among market participants in an opaque market structure. For this, we assume that our units of observation, highly sophisticated financial market participants, are fully

[3] The modern literature on information aggregation is too large to do justice to here. Important contributions have focused on auctions (Pesendorfer and Swinkels (1997); Kremer (2002)), decentralized trading (Gale (1986), Golosov et al. (2014)), asset design (Ostrovsky (2012)) or the trade-off between market size and information heterogeneity (Rostek and Weretka (2012)).

rational Bayesian agents. We then use this structural assumption to derive measurement devices for uncertainty based on market participants' model-implied beliefs. An additional novel aspect of our empirical approach is the focus on strategic uncertainty. Here, the structural approach is particularly useful as data on market participants' higher-order beliefs are typically not available. Hortaçsu and Kastl (2012) and Hortaçsu et al. (2018), for example, use model-implied beliefs derived from a structural estimation to gauge the strategic value dealers derive from being able to observe client demand in Treasury auctions. Similarly, Boyarchenko et al. (2019) use a calibrated model to perform counterfactual informational experiments in the US Treasury market to evaluate the welfare implications of different order flow information-sharing arrangements among dealers and clients. However, the source of strategic information in these models is order flow information rather than price data. More generally, we see the counterfactual experiments we perform on the market's information structure as an illustration of the usefulness of a structural approach for empirical work on information design (Bergemann and Morris (2019)).

This paper also advances the understanding of the role financial benchmarks have for market functioning. Duffie et al. (2017) show how benchmarks can reduce informational asymmetries in search markets and thereby increase the participation of less-informed agents. Here, we focus on the informational content of the benchmark itself. This paper is, to our knowledge, the first to provide an empirical evaluation of the informational properties of a consensus pricing mechanism.[4] While the importance of benchmarks for financial markets is widely appreciated, the attempted manipulation of major interest rate benchmarks has led to a regulatory push to base benchmarks on transaction prices or firm quotes rather than expert judgment (IOSCO (2013)). However, in illiquid markets and during crisis times, this might not always be possible. During the COVID-19 turmoil in March 2020, for example, three out of four candidate forward-looking term rate benchmark providers were unable to publish benchmarks during a three-day period due to the lack of transactions data (Risk.net). More generally, this paper illustrates the informational value of non–transaction based price information for decentralized financial markets. Previous work in this area has focused on information aggregation mechanisms predominately used in centralized stock markets, in

[4]Many important financial benchmarks are consensus prices. It is also employed by information providers, such as Bloomberg, to calculate "generic prices" for a wide range of financial products.

particular pre-opening prices (Biais et al. (1999), Cao et al. (2000)) and opening auctions (Madhavan and Panchapagesan (2000)).

The plan of the paper is as follows. Section 2 provides a brief explanation of the option market structure and the Totem consensus pricing service and presents the data. Section 3 develops the structural model of learning from consensus prices. Section 4 presents the structural estimation of the model and discusses identification and robustness. Section 5 explains our approach to measuring valuation and strategic uncertainty and presents results. Section 6 concludes.

2 Market structure and data

We start by providing a short overview of the structure of the options markets. We then introduce the Totem consensus pricing service and explain the submission process. At the end of the section, we provide some stylized facts of the consensus price data that motivate our structural modelling approach.

2.1 Options market structure

Options on the S&P 500 index are arguably the central derivatives contracts for the US stock market. Option prices contain rich information on market participants' beliefs about future stock market movements and risk premia.[5] The VIX index, a popular measure of risk perception in financial markets, is based on S&P 500 option prices. The dominant market structure for options trading depends on the terms of the contract. Option contracts with times-to-expiration of less than 6 months and strike prices close to the current index value (moneyness close to 100) are typically traded via limit order books on options exchanges such as the Chicago Board Options Exchange (CBOE). Price quotes, transaction prices and volumes are fully transparent and are available to all market participants. For options with

[5] An option contract on an asset gives the owner the right (but not the obligation) to buy (call option) or sell (put option) the asset. The time-to-expiration of (European style) contracts specifies the date at which the option can be exercised; the strike price specifies the price at which the asset can be bought or sold. The strike price is often expressed as a ratio to the current price of the asset times 100. This is also called the moneyness of the option.

more extreme contract terms, the dominant market structure is OTC trading. Figure 1 displays the average on-exchange trading activity for S&P 500 index option over the period 2002 to 2015 for contract terms covered in this paper. On-exchange trading activity is decreasing with the time-to-expiration and the extremeness of the strike price of an option. For option contracts with times-to-expiration of more than 3 years, trading is almost exclusively OTC.[6]

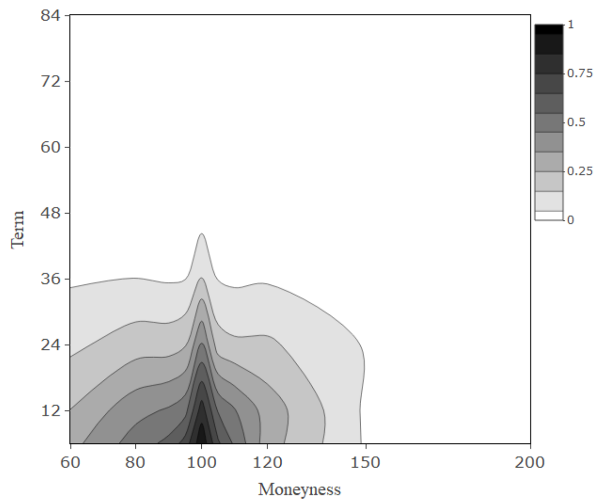


Figure 1: This figure displays the percentage of trading days on which an S&P500 option contract has an aggregate trading volume of at least 10 contracts on US options exchange according. Volumes for exchange-traded contracts in proximity to a given Totem contract are aggregated and mapped to the corresponding moneyness/term combination. The x-axis gives the moneyness of the contracts, the y-axis times-to-expiration in months. The sample period is 2002 to 2015 (Data: OptionMetrics).

The OTC segment of the options market is centred around a network of dealers. These are typically large investment banks that act as market-makers and trade with each other and with so-called clients: hedge funds, asset managers, insurance companies, and pension funds that need to manage portfolio risk or intend to establish speculative positions. In terms of clientele, the market segment with times-to-expiration below one year is typically dominated by hedge funds trading short-term stock market volatility. The one- to three-year segment tends to be the domain of "real money," asset managers such as BlackRock that use options in their portfolio insurance strategies. Client demand in the market segment

[6]Clearing house data (see, e.g., OOC) shows that in overlapping regions, traders often choose an OTC over an on-exchange trade. This can be because of greater flexibility in contract terms, lower trading fees, or market impact considerations. Large trades can be difficult to hedge if the trade is publicized. Here the transparency that comes with on-exchange trading is undesirable.

with times-to-expiration beyond 3 years tends to come from pension funds and life insurance companies that have long-term commitments linked to the evolution of the stock market. In the OTC market, trades are negotiated bilaterally, often over the phone, by email or instant messaging. Both transaction price and volume remain proprietary information of the two parties involved in the trade.[7] Rather than having to rely on pricing models to hedge option exposures, dealer banks typically prefer to conduct offsetting trades with each other in the inter-dealer segment of the options market. Hence, when trading with clients, a dealer bank not only has to form a view on the fundamental drivers of option values. It also has to consider the valuations of other dealer banks with whom it can enter into offsetting trades. In online Appendix 8.1 we develop a stylized model of this market structure to illustrate the value of information for dealer banks.

2.2 Consensus Price Data

The empirical analysis is based on data from the main consensus pricing service for the OTC derivatives market, IHS Markit's Totem service. The service started in February 1997 with 6 major OTC derivatives dealers. Since then, Totem has become the leading platform for OTC consensus price data, with around 120 participants and a coverage of all major asset classes and types of derivatives contracts. In this paper, we focus on the consensus prices for call and put options on the S&P 500 index. We have access to the full history of Totem contributors' price submissions. The individual institutions are anonymized, but we can track each institution's submissions over time and across contracts. We restrict our sample to the period December 2002 to February 2015 to achieve a consistent set of option contracts and a stable group of submitting institutions. Table 1 reports the set of option contracts we consider (by time-to-expiration and moneyness) as well as the average number

[7] Some dealers run proprietary electronic trading platforms on which they post price quotes. In 2010, various electronic trading platforms introduced request-for-quote systems to further increase pre-trade price transparency. The regulatory reforms after the financial crisis have also introduced mandatory post-trade reporting to trade repositories for all OTC derivatives transactions. But these are regulatory data and not available to market participants. Another source for price and volume information in OTC markets is central counterparties (CCPs). However, unlike for interest rate and credit derivatives, OTC equity derivatives trades are currently not subject to a central clearing mandate. The current proportion of OTC equity derivatives trades that is centrally cleared is negligible (see Financial Stability Board (2018)).

of institutions submitting price estimates for a given contract over our sample period.[8]

Table 1: Average number of submitters

	moneyness										
term	60	80	90	95	100	105	110	120	130	150	200
6	27	31	31	31	31	31	31	31	29	27	·
12	27	30	30	30	30	30	30	30	28	27	·
24	27	30	30	30	30	30	30	30	28	26	19
36	26	29	29	29	29	29	29	29	27	26	18
48	26	29	29	29	29	29	29	29	26	25	18
60	25	28	28	28	28	28	28	28	26	25	18
84	24	25	25	25	25	25	25	25	24	23	17

This table provides the average number of dealer banks that submit to a given S&P 500 options contract. The numbers are based on accepted submissions for the given date. The data sample is from December 2002 to February 2015.

The consensus pricing process

The Totem pricing service typically operates at a monthly frequency. At the end of a month, all submitters are asked to provide their best estimates of the mid-quotes for the set of derivatives contracts to which they contribute. In addition to their estimate of the contract price itself, this includes other auxiliary information, such as discount factors, dividend yields, and the price of the underlying asset. Totem indicates the precise time at which valuations are to be made on the so-called valuation day.

Manipulation incentives for consensus prices of OTC derivatives are generally weaker than for benchmark interest rates, such as LIBOR, that are compiled using a similar methodology. Unlike for interest rate benchmarks, there are no financial products that are indexed to OTC derivatives consensus prices. Hence, changes in consensus prices do not immediately impact an institution's profits and losses from other investments. Furthermore, the Totem consensus pricing service has significantly more submitters than interest rate benchmarks. Even for the most extreme contract on average 17 institutions contribute prices, which makes manipulation of the consensus price strategically challenging. Nevertheless, Totem uses a

[8]For contracts with time-to-expiration of 6 and 12 months, we exclude the contracts with a moneyness of 200. For these contracts, prices are close to zero and crucially depend on the numerical precision used by Totem submitters when reporting prices. Additionally, the inversion of the prices to Black-Scholes implied volatilities can become numerically unstable.

range of formal and informal procedures to discourage price manipulation and incentivize high-quality price submissions. For the S&P 500 option service, each submitter is obligated to contribute to the contracts with time-to-expiration of 6 months and moneyness, expressed as the ratio of the option's strike price to the current index level, between 80 and 120. 'No-arbitrage" arguments between contracts allow for consistency checks between contracts. In case of doubt, additional private conversation between submitters and IHS Markit employees (often former market participants) can take place to gather additional information on individual prices and market conditions. Price submissions that are deemed of low quality do not enter the consensus price calculation, and the submitting institution does not receive the consensus price for that submission period. This serves as a formal punishment mechanism for low-quality submissions.

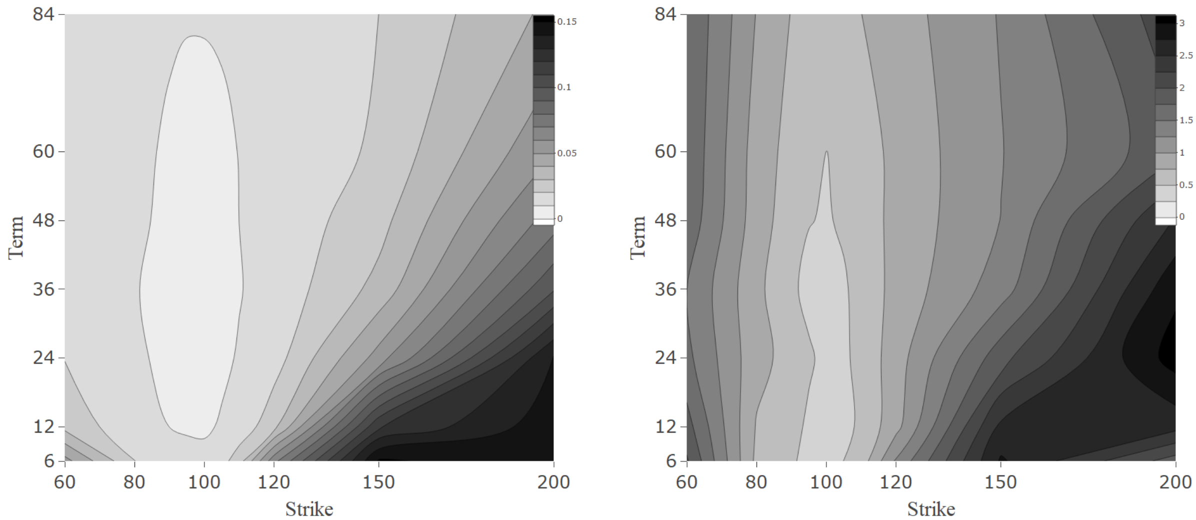
Accepted price submissions are then used to calculate consensus prices, one for each derivatives contract. The highest and lowest accepted price are dropped from the sample before the mean is calculated. Totem provides contributors with the new consensus prices within 5 hours of their initial price submissions. The consensus price for a given option contract is the arithmetic mean of the accepted price estimates. Totem submitters only receive aggregated data from the service. They do not observe data on other institutions' individual submissions. We provide a detailed description of the submission process and the quantities submitted in online Appendix 8.2.

Valuation differences among dealers

To provide a sense of the cross-sectional dispersion in Totem submitters' option valuations, the left panel of Figure 2 depicts the cross-sectional standard deviation of price submissions averaged over the sample period.[9] There is considerable variation in the dispersion of dealers' submitted prices across the contract space. It is highest for short-dated options with extreme strike prices. For a given time-to-expiration, the dispersion is lowest for strike prices close to the current index level, that is a moneyness of 100. The price dispersion across dealers tends to decrease with time-to-expiration. These cross-sectional differences are economically

[9] Throughout the paper, we express option prices in terms of Black-Scholes implied volatilities (IVs). This is the market convention for quoting option prices and facilitates the comparison of option prices across times-to-expiration and strike prices.

meaningful; they are of similar magnitude to bid-ask spreads observed on option exchanges in regions where OTC and on-exchange trading overlaps, but they display a low level of correlation with these bid-ask spreads over time, as can be seen in Figure 7 of the online Appendix.



(a) Cross-sectional dispersion

(b) Half-lives

Figure 2: The **left figure** displays the time-series average of the cross-sectional standard deviation of submitters' implied volatility estimates for S&P 500 OTC index options. The **right figure** presents estimates for the half-life (in months) of the individual deviations from the contemporaneous consensus price. Half-lives derive from AR1 coefficients estimates of (1): $\hat{t}^c = \log(0.5)/\log(\hat{\beta}^c)$. The estimates are from a pooled ordinary least squares regression. The y-axes show times-to-expiration in months, the x-axes moneyness. The sample period is December 2002 to February 2015.

The right panel of Figure 2 shows the persistence of individual dealers' deviations from the consensus price. For each option contract, we estimate the following AR(1) regressions to quantify this persistence:

$$p^c_{i,t} - p^c_t = \beta^c \left(p^c_{i,t-1} - p^c_{t-1}\right) + \epsilon^c_{i,t}. \tag{1}$$

Here $p^c_{i,t}$ is institution i's price submission for contract c in period t and p^c_t is the corresponding consensus price. The right panel of Figure 2 reports the estimated β coefficients expressed as half-lives, i.e., the number of months it takes to close half of the gap between an individual dealer's price estimate and the consensus price. Dealers' deviations from consensus are persistent for all contracts. The U-shaped persistence pattern in moneyness partially mirrors the cross-sectional dispersion in the left panel of Figure 2.

From these statistics of the raw data, we draw some preliminary observations that guide our structural modelling. First, all dealers are asked to provide their best estimate for the mid-quote of a given contract, i.e. a market-wide price. If all dealers had access to the same information and used the same models, they should all provide the same price estimate. In this paper we abstract from model disagreement or model uncertainty and assume that dealers form expectations by updating beliefs using a model that itself is common knowledge. Under this interpretation of the data, the observed cross-sectional dispersion necessarily reveals informational frictions in the OTC market. Furthermore, these frictions vary across market segments.

Second, these informational frictions have to derive from dealer banks' private information. Imperfect information that is observed by all dealer banks does not induce cross-sectional dispersion. However, the cross-sectional dispersion alone cannot identify the precision of private valuation information, as both very precise and very imprecise private information imply low cross-sectional dispersion. This illustrates the conceptual problem of using the cross-sectional dispersion of the raw data for the measurement of informational frictions.

Last, if the consensus price perfectly aggregates dispersed information, all bank dealers should have the same expectation about the current mid-quote after observing the current consensus price. Any deviation from the consensus price in the next period has to be the result of new private information. But this implies that a dealer's past relative position to the consensus price has no predictive power for its future relative position; individual deviations from consensus cannot be persistent. This is clearly rejected by the data. The positive persistence points to imperfect information aggregation and, consequently, long-lived private information at the level of individual dealers.

3 A Model of Consensus Pricing

We model the consensus pricing process as a social learning problem. Dealers learn about a time-varying fundamental value from private signals and the consensus price. We derive the consensus price process as an equilibrium outcome of the model and show what structure this imposes on dealers' belief updating dynamics.

3.1 The model

A large number of dealers, modelled as a continuum indexed $i \in (0,1)$, participate in a consensus pricing service. At discrete submission dates $t = \{..., -1, 0, 1, ...\}$ each dealer i submits its best estimate for the current value of an option, θ_t, to the service. θ_t is latent and follows an AR(1) process,

$$\theta_t = \rho\,\theta_{t-1} + \sigma_u\, u_t \;\text{ with }\; u_t \sim N\left(0,1\right), \tag{2}$$

and $-1 < \rho < 1$. We do not explicitly model the economic forces responsible for the variation in fundamental option values. A possible interpretation is based on a demand-based option pricing model in the spirit of Gârleanu et al. (2009).[10]

At each submission date t, dealers observe two signals about the fundamental option value, a noisy private signal and the consensus price. Dealer i's noisy private signal is

$$s_{i,t} = \theta_t + \sigma_\eta\, \eta_{i,t} \;\text{ with }\; \eta_{i,t} \sim N\left(0,1\right), \tag{3}$$

where $1/\sigma_\eta^2$ measures the precision of the private signal. The precision does not depend on i, that is all dealers receive signals of equal quality. The private signal structure implies that dealers are able to infer the current fundamental value if they pool their private informa-

[10]Under a demand-based interpretation, changes in the fundamental value derive from time-varying client demand that is satisfied by risk-averse broker-dealers. u_t is an aggregate demand shock for options with a given strike price and time-to-expiration. The AR(1) process for the fundamental value can be obtained in a setting in which the underlying asset follows a geometric Brownian motion, as is assumed in the Black-Scholes model (Black and Scholes (1973)). All time variation in fundamental option prices (expressed in terms of implied volatilities) is then driven by client demand shocks that cannot be perfectly hedged using the underlying, for example because of an inability to trade continuously. We provide a derivation of this setup in online Appendix 8.3.

tion; there is no aggregate noise in private signals. Taking a demand-based pricing view for our OTC market setting, this is a natural assumption. Trades with clients are the primary source of a dealer's private information. These trades provide an imperfect signal of aggregate demand conditions. Pooling client demands across dealers reveals current aggregate demand.

In addition to the private signal, each dealer observes last period's consensus price p_{t-1}. The timing of the consensus pricing process under which dealers only obtain the consensus price once they have submitted their estimates is a key difference to standard rational expectations equilibrium (REE) models. As the consensus price is a signal of the market's past state, it does not supplant a dealer's noisy private signal even if it is perfectly revealing. We model the current consensus price p_t as a noisy signal of the average estimate of θ_t across dealers. Dealer i's information set at the time of period t's price submission consists of the (infinite) history of previous consensus prices and the private signals that i has observed up to period t, that is $\Omega_{i,t} = \{s_{i,t}, p_{t-1}, \Omega_{i,t-1}\}$. All dealers submit their best estimate of θ_t to the service. For each dealer, we take this to mean its conditional expectation of θ_t given $\Omega_{i,t}$. We denote this conditional expectation by

$$\theta_{i,t} = \mathbb{E}\left(\theta_t | \Omega_{i,t}\right),$$

and the corresponding cross-sectional average across dealers by

$$\bar{\theta}_t = \int_0^1 \theta_{i,t}\, di.$$

The consensus price itself is a noisy signal of this average expectation,

$$p_t = \bar{\theta}_t + \sigma_\varepsilon\, \varepsilon_t \;\text{ with }\; \varepsilon_t \sim N\left(0, 1\right). \tag{4}$$

We do not specify dealers' preferences, which would determine why they value the consensus price information. Certain preference specifications could create an incentive to strategically manipulate the consensus price, for example in order to experiment (e.g. Brancaccio et al. (2020)) or to gain a competitive advantage. However, given the assumption of a continuum of dealers (and mild technical restrictions on admissible submissions), no single submitter can influence the consensus price. Hence, asking dealers to submit their best estimate of θ_t

is compatible with their incentives.[11]

Modelling the consensus price as a noisy public signal of average expectations is motivated by two considerations. First, as previously discussed, Totem eliminates the lowest, the highest, and problematic price submissions from the consensus price calculations. Hence, the consensus price itself does not exactly correspond to the average submission. Second, while we assume a continuum of dealers, we want to allow for the possibility that the consensus price does not fully reveal the average expectation and, consequently, last period's fundamental value.[12] Knowing last period's fundamental value rules out long-lived private information. But such long-lived private information is needed to capture the persistence of the deviations of individual price submissions from the consensus price, a feature we observe in the data.

A remaining question is how commonly observed exchange-based option price data, for example prices listed on the Chicago Board Options Exchange (CBOE), fit into the above framework. A way to think about such data within our model is to interpret a dealer's submission, $\theta_{i,t}$, as the difference between the dealer's best estimates of the current fundamental value of the OTC contract and the current corresponding exchange-based price for the option. If no exchanged-based prices are available for a given Totem contract, an industry-standard, and hence commonly known, option pricing model calibrated to exchange-based prices for other contracts could be used instead. To map our data into the model, a dealer i's Totem submission and the corresponding consensus price are then $\theta_{i,t}$, respectively p_t, plus the price for the contract as given by exchange-based trading activity. An important assumption under this interpretation of the data is that exchange-based prices do not contain any information about u_t, $\eta_{i,t}$, and ε_t that is not already contained in a dealer's history of private signals and past consensus prices. In that sense, these shocks should be seen as specific to OTC market conditions, which is consistent with the stated purpose of the Totem consensus pricing service.[13]

[11] Raith (1996) gives a theoretical analysis of the incentives for competitive firms to participate in (truthful) information-sharing arrangements. In online Appendix 8.1 we develop a stylized model that illustrates the value of consensus pricing information for dealers that operate in an OTC market.

[12] We estimate σ_ε and allow the parameter to be 0. This implies that full revelation is a possible outcome.

[13] In our estimation, we treat exchange-based prices as latent and use contract fixed effects. Explicitly incorporating exchange-based price data for option contracts that are exclusively OTC-traded would require specifying the option pricing model that is used by all dealers and taking a stand on how to calibrate this

3.2 Learning from consensus prices

In order to characterize dealer i's submission to the consensus pricing service, we need to calculate the dealer's conditional expectation $\mathbb{E}\left(\theta_t|\Omega_{i,t}\right)$. Its information set $\Omega_{i,t}$ depends on all other dealers' submissions via the consensus price process p_t. This information set is endogenous as p_t is both an input and an output of the joint learning process of the consensus pricing participants. As first pointed out by Townsend (1983), signal extraction problems in which signals are equilibrium variables, such as prices, typically do not admit representations in which a finite number of variables can summarizes the current state of the system. For very restrictive settings, frequency domain techniques have been successfully employed to obtain exact finite state space representations, e.g. Kasa (2000). However, a popular direction of attack is truncation, i.e. to show that the original problem is well approximated by a finite state space even if the actual solution requires an infinite number of states (e.g. Sargent (1991), Lorenzoni (2009), Huo and Pedroni (2020)).

This is the approach taken here. We adopt an iterative algorithm previously used in Nimark (2014) and Barillas and Nimark (2017) to solve our filtering problem. The algorithm works as follows:

1. Start with any covariance-stationary process (p_t^0) that lies in the space spanned by linear combinations of current and past aggregate shocks (u_t) and (ε_t).

2. This consensus price process (p_t^0) yields information sets for all i and t defined recursively by $\Omega_{i,t}^0 = \{s_{i,t}, p_{t-1}^0, \Omega_{i,t-1}^0\}$.

3. Given information set $\Omega_{i,t}^0$, dealer i can compute the conditional expectation $\mathbb{E}(\theta_t|\Omega_{i,t}^0)$ for period t under the assumed stochastic process for (p_t^0).

4. Averaging the expectations across submitters yields a new consensus price process

$$p_t^1 = \int_0^1 \mathbb{E}(\theta_t|\Omega_{i,t}^0)\, di + \sigma_\varepsilon\, \varepsilon_t \ \text{ for all } t.$$

5. If the distance (in m.s.e.) between (p_t^0) and (p_t^1) is smaller than some pre-specified

model to observed prices.

stopping criterion, stop. Otherwise, go to step 2 with (p_t^1) as the new consensus price process and so on.

For any initial choice of (p_t^0), the sequence of price processes $\{(p_t^n)\}_n$ converges (in m.s.e.) to a unique limit process (p_t), the solution of the original filtering problem, when the integral in step 4 is a contraction on the space of covariance-stationary price processes. Starting with the initial guess $p_t^0 = \theta_t + \sigma_\varepsilon \varepsilon_t$ allows the problem to be solved by a sequential application of the Kalman filter. It also provides an upper bound on the approximation error if the algorithm is stopped after a finite number of steps.

After n steps, the equilibrium learning dynamics are approximated by a linear state-space system with an $n+1$ dimensional state vector x_t. The first and second element of x_t are the fundamental value θ_t and the cross-sectional average expectation $\bar{\theta}_t$, respectively.[14] The state evolves according to

$$x_t = M x_{t-1} + N v_t \quad \text{with} \quad v_t = (u_t, \varepsilon_{t-1})^\intercal, \quad v_t \sim N(0, I_2). \tag{5}$$

The matrices M and N are known functions of the model parameters, namely $\{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$. A dealer's signals in period t can be expressed as noisy observations of the state,[15]

$$s_{i,t} = e_1^\intercal x_t + \sigma_\eta \eta_{i,t} = \theta_t + \sigma_\eta \eta_{i,t},$$
$$p_{t-1} = e_2^\intercal x_{t-1} + \sigma_\varepsilon \varepsilon_{t-1} = \bar{\theta}_{t-1} + \sigma_\varepsilon \varepsilon_{t-1}.$$

The two signals can be written in vector form as

$$z_{i,t} = D_1 x_t + D_2 x_{t-1} + B \epsilon_{i,t},$$

with $z_{i,t} = (s_{i,t}, p_{t-1})^\intercal$ and $\epsilon_{i,t} = (v_t^\intercal, \eta_{i,t})^\intercal$.

[14] The k-th element of x_t is the cross-sectional average of dealers' k-th order expectation of θ_t given their information in period t. The average k-th order expectation is defined recursively as $\theta_t^{(k)} = \int \mathbb{E}(\theta_t^{(k-1)} | \Omega_{i,t}) \, di$ with $\theta_t^{(1)} = \bar{\theta}_t$. Appendix 7.1 provides a more detailed explanation of these higher-order expectations and the solution algorithm.

[15] Here, $e_n^\intercal$ is a vector with 1 in the n^{th} position, 0 otherwise.

We can now use the Kalman filter to obtain dealer i's beliefs about θ_t and $\bar{\theta}_t$, the first two elements of x_t, given the information in $\Omega_{i,t}$. The linear-normal structure of the state-space system implies that dealer i's beliefs are normally distributed,

$$x_t \mid \Omega_{i,t} \sim N\left(x_{i,t}, \Sigma^p\right), \tag{6}$$

where the conditional expectations about the state evolve according to

$$x_{i,t} = Mx_{i,t-1} + K\left(z_{i,t} - \left(D_1 M + D_2\right) x_{i,t-1}\right), \tag{7}$$

and K is a $(n+1) \times 2$ dimensional matrix of Kalman gains. Here K and the covariance matrix of dealers' beliefs Σ^p are known functions of the model parameters.[16]

4 Estimation

We estimate the model's parameter vector $\phi = \{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$ separately for each contract. This contract-by-contract approach is consistent with the above model and we can prove that the structural parameters of the model are identified. Furthermore, estimating each contract separately allows us to verify that coefficients estimates do not vary substantially across adjacent contracts. This is a reasonable a priori assumption in our context and provides an additional sanity check for the estimation results. A joint estimation, on the other hand, would require imposing a correlation structure on fundamental shocks and signals across contracts. As not all participating dealer banks submit to all contracts, modelling compositional effects would add an additional layer of complexity to a joint estimation framework.

For a given contract, our data consists of the panel of Totem price submissions by individual dealers and the corresponding consensus price. We denote by $\iota_t \subset \{1, 2, .., S\}$ the set of dealers active in t. S is the total number of distinct dealers that have submitted to Totem over the course of our sample period. The time series of submissions is given by $(\mathbf{p}_t)_{t=1}^{T}$, where

[16] Given the infinite history of past signals, the covariance matrix Σ^p and the matrix of Kalman gains K are not time dependent. Also, Σ^p and K are not dealer-specific as dealers are symmetrically informed. They all receive signals of the same quality. Superscripts, here p, are used to index information structures. Later, we modify the information structure of the market in counterfactual experiments.

$\mathbf{p}_t = (p_{i,t})_{i \in \iota_t}$ is a $|\iota_t|$-dimensional vector consisting of the individual period t consensus price submissions. Our data set for a given contract, $(\mathbf{y})_{t=1}^{T}$, then consists of the time series of dealers' price submissions for this contract and the corresponding consensus price, i.e. $\mathbf{y}_t = (p_t \, , \mathbf{p}_t)^{\mathsf{T}}$.[17]

4.1 Likelihood function and estimation

To estimate the model for a given contract, we cast it into state-space form. The panel of individual price submissions and the time series of consensus prices constitute the available observations of the system. Based on Section 3, the latent state space has the following dynamics:

$$x_t = M(\phi)\, x_{t-1} + N(\phi)\, v_t \quad , \quad v_t \sim N(\mathbf{0}, I_2),$$

where $v_t = (u_t \; \varepsilon_{t-1})^{\mathsf{T}}$. $M(\phi)$ and $N(\phi)$ are obtained by employing the previously explained solution algorithm for a given parameter vector ϕ. We assume that dealer i's price submission for period t is its conditional expectation of θ_t, i.e. $p_{i,t} = \theta_{i,t}$.

Dealer i's conditional expectations of the current state x_t are updated as follows,

$$x_{i,t} = M(\phi)\, x_{i,t-1} + K(\phi) \left[\begin{pmatrix} s_{i,t} \\ p_{t-1} \end{pmatrix} - (D_1\, M(\phi) + D_2)\, x_{i,t-1} \right]. \tag{8}$$

In the estimation, we treat a dealer i's private signal $s_{i,t}$ as a latent variable. It is observed by the dealer but not by the econometrician. The noise in the private signal is assumed to be uncorrelated across submitting dealers and time. The consensus price p_t is observed by both the dealers and the econometrician. The econometrician only observes the first element of dealer i's conditional expectations $x_{i,t}$ which is $\theta_{i,t}$, dealer i's conditional expectation of the current fundamental value. We assume that it is this expectation that the dealer submits to the Totem service.

[17] To be precise, we take the natural logarithm of the implied volatility (IV) corresponding to, respectively, dealer i's price submission and IHS Markit's consensus price for each time t. We then subtract the time series average of the logged consensus IV series from all individual logged IV time series and the consensus IV series itself to remove a contract specific level. The resulting objects are series of $p_{i,t}$ for each dealer j and the consensus series p_t.

Equation (8) provides us with a disciplined way of modelling the belief updating dynamics at the level of the individual dealer. It illustrates both the usefulness of the model to impose structure on belief data and the importance of observing a time series of beliefs at the individual level to estimate dynamic social learning models. A more simplistic approach that estimates belief-updating dynamics using only first-order expectations without taking into account the importance of higher-order expectations in the filtering problem will suffer from an omitted variable bias.

Given the linearity of the above system and the joint normality of all shocks, the likelihood function for the observed data $(\mathbf{y}_t)_{t=1}^{T}$ can be derived using the Kalman filter. We obtain estimates for the parameter vector ϕ using MCMC methods with diffuse priors.[18] We constrain $\sigma_u, \sigma_\varepsilon$, and σ_η to be positive and $0 < \rho < 1$. For each contract we run chains of length 100.000 with the Metropolis-Hastings algorithm and disregard the first 10.000 draws. We subsequently pick every 10^{th} draw to construct the posterior distribution of the parameters. Appendix 7.2 provides a detailed derivation of the filter for the above model. Table 3 in the Appendix reports parameter estimates and standard errors for $\rho, \sigma_u, \sigma_\varepsilon$, and σ_η for all contracts.

4.2 Identification

The Appendix 7.3 provides a formal proof of identification for the model. Here, we give a short summary of which moments of the data help us to identify the structural parameters of the model. The time-series variance of the differences between p_t and cross-sectional average of submission identifies σ_ε. The speed at which individual deviations from the average submission mean-revert determines the weight dealers put on their prior expectations (as opposed to weight put on news in the current signal and consensus price). Knowing this weight allows us to isolate changes in price submissions that are due to new information a dealer received in a given period. As these news are linked to the current fundamental, the autocorrelation of these expectation updates that have been "cleaned" of prior expectations allow us to identify ρ, the persistence in the fundamental value process. The weight dealers

[18]While we use Bayesian methods to obtain the posterior parameter distributions, using diffuse priors implies the estimates can also be thought of as classical maximum-likelyhood point estimates with the standard deviations corresponding to classical standard errors.

put on their prior depends on how persistent the fundamental is and how high the quality of their new information is, i.e. the signal-to-noise ratio of their signals. Having identified ρ, we can now identify this signal-to-noise ratio from the weight dealers put on their prior expectations. The signal-to-noise ratio depends on the variance of the fundamental shocks, σ_u^2, and the precision of private signals and the consensus price as determined by σ_η and σ_ε. We have already identified σ_ε. The relative weight dealers put on the consensus price as opposed to the private signal depends on the relative precision of these two signals. This allows us to identify σ_η and, finally, σ_u from the signal-to-noise ratio.

4.3 Model fit and robustness checks

To judge how well the model fits the data, we compare the model-implied cross-sectional dispersion of price submissions and the time-series volatility of the consensus price to their empirical analogues. In the online Appendix, the ratio of the model-implied cross-sectional standard deviation and the empirical cross-sectional standard deviation are displayed. This ratio for the different contracts is between 0.909 and 1.125, which implies that the model is able to reproduce the size of the cross-sectional dispersion for the different contracts. We also do not find a statistically significant difference between the model-implied and the empirical consensus price volatility.

The sample period includes two peculiar periods: the low volatility period from 2002 to 2006 and the Great Recession from 2007 to 2010. The parameter estimates for these periods might be very different. However, we find that results do not change if we consider these two periods separately. Another potential split is that of dealers that participate for a limited time frame and routine dealers. We therefore re-estimate the model excluding dealers who have submitted for less than 40% of the total sample period. The parameter estimates are comparable to the results for the entire sample. Including only the 'routine' dealers makes the contrast between the at-the-money (ATM) and the out-of-the-money (OTM) options slightly larger. The estimation results for these three additional data treatments are reported in the online Appendix.

5 Results

This section derives model-implied measures of valuation and strategic uncertainty based on dealers' posterior beliefs. The estimates of the structural model parameters provide us with estimates for these uncertainty measures and we show how dealer uncertainty varies across market segments. Counterfactual experiments on the option market's information structure allow us to quantify the informational value of the consensus prices for dealer banks.

5.1 Valuation and strategic uncertainty

At the time of the period t consensus price submission, dealer i's posterior beliefs about the current fundamental value of a contract, θ_t, and the cross-sectional average expectation of this value, $\bar{\theta}_t$, are jointly normally distributed with

$$\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} \mid \Omega_{i,t} \sim N\left(\begin{pmatrix} \theta_{i,t} \\ \bar{\theta}_{i,t} \end{pmatrix}, \begin{pmatrix} (\sigma_{11}^p)^2 & \sigma_{12}^p \\ \sigma_{12}^p & (\sigma_{22}^p)^2 \end{pmatrix} \right). \tag{9}$$

The dealer's conditional expectations about θ_t and $\bar{\theta}_t$ are updated according to

$$\theta_{i,t} = \rho\, \theta_{i,t-1} + k_s\ (s_{i,t} - \rho\, \theta_{i,t-1}) + k_p \left(p_{t-1} - \bar{\theta}_{i,t-1} \right), \tag{10}$$

$$\bar{\theta}_{i,t} = m_2 \cdot x_{i,t-1} + \bar{k}_s\ (s_{i,t} - \rho\, \theta_{i,t-1}) + \bar{k}_p \left(p_{t-1} - \bar{\theta}_{i,t-1} \right). \tag{11}$$

The covariance matrix in (9) corresponds to the top left 2×2 sub-matrix of Σ^p given in (6). The parameters k_s and k_p in (10) are the Kalman gains for the private signal and the consensus price, respectively. The Kalman gains are the weights a dealer puts on the "news" contained in the signals when updating expectations about the fundamental value θ_t. They correspond to the first row of K in (7). Similarly, $\bar{k}_s$ and $\bar{k}_p$ in (11) are the Kalman gains for the private signal and consensus price for the average expectation $\bar{\theta}_t$. They correspond to the second row of K.

Our measures of valuation uncertainty and strategic uncertainty are the variance of a dealer's forecast errors for the fundamental value, $\theta_{i,t} - \theta_t$, and the variance of its forecast error for the average expectation, $\bar{\theta}_{i,t} - \bar{\theta}_t$, at the time of the consensus price submission. These variances are $(\sigma_{11}^p)^2$, respectively $(\sigma_{22}^p)^2$, as given in (9). The correlation between the two forecast errors, $\rho_{12} = \sigma_{12}^p/(\sigma_{11}^p \sigma_{22}^p)$, is a natural measure for what Angeletos and Pavan (2007) call the "commonality of information": higher correlations imply that different dealers interpret new valuation information in a similar way.

We start by developing some intuition for the relationship between valuation uncertainty, strategic uncertainty and the commonality of information before presenting the estimation results. To do so, we split up the expectation updating for θ_t into two steps. First, the dealer updates its expectations about θ_{t-1} after observing the consensus price p_{t-1}. Call this updated expectation $\theta_{i,t-1}^+$. Next, the dealer receives the private signal $s_{i,t}$ and updates its expectation to $\theta_{i,t}$. This is what the dealer submits to the Totem service. We can therefore decompose the submission as follows,

$$\theta_{i,t} = (1 - k_s)\rho\, \theta_{i,t-1}^+ + k_s\, s_{i,t}.$$

Averaging across dealers and noting that idiosyncratic noise cancels out, the average expectation across dealers can now be expressed as

$$\bar{\theta}_t = (1 - k_s)\rho\, \bar{\theta}_{t-1}^+ + k_s\, \theta_t.$$

Finally, taking expectations of the above expression conditioning on $\Omega_{i,t}$ and subtracting the original equation allows us to link the forecast errors for θ_t and $\bar{\theta}_t$,

$$\bar{\theta}_{i,t} - \bar{\theta}_t = (1 - k_s)\left[\mathbb{E}(\rho\, \bar{\theta}_{t-1}^+ | \Omega_{i,t}) - \rho\, \bar{\theta}_{t-1}^+\right] + k_s\left(\theta_{i,t} - \theta_t\right).$$

The above expression shows that the forecast error for $\bar{\theta}_t$ is a weighted sum of the forecast error for θ_t and the forecast error for the average prior expectation about θ_t before observing the private signal in period t. Dealer i's forecast errors for $\bar{\theta}_t$ and θ_t are perfectly correlated, i.e. $\rho_{12} = 1$, if the dealer knows the average expectation $\bar{\theta}_{t-1}^+$. In our model this can only happen if the consensus price perfectly aggregates all dispersed information. In that case, all dealers have the same expectation after observing the consensus price, namely $\bar{\theta}_{t-1}^+ = p_{t-1}$, and the average posterior expectation is given by $\bar{\theta}_t = (1 - k_s)\rho\, p_{t-1} + k_s\, \theta_t$. It follows that dealers are less uncertain about $\bar{\theta}_t$ than about θ_t as $\sigma_{22}^p = k_s^2\, \sigma_{11}^p$ and $|k_s^2| < 1$.

If dealers are uncertain about the average expectation $\bar{\theta}^{+}_{t-1}$, this simple relationship no longer holds. (10) can be written as

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k_s\,s_{i,t} + k_p\,p_{t-1} + k_p\,\left(\theta_{i,t-1} - \bar{\theta}_{i,t-1}\right) \quad \text{with} \quad k = k_s + \frac{k_p}{\rho}. \tag{12}$$

We see that if dealers put strictly positive weight on their individual prior expectations, that is $1-k>0$, their expectations are partially dependent on their individual history of private signals. Hence, dealers do not return to a common market perception after observing the consensus price. A lack of common perspective on past market conditions partially feeds into uncertainty about current market conditions as given by $\bar{\theta}_t$.

5.1.1 Price versus private information

Figure 3 displays estimates of the Kalman gains for the private signal and consensus price for both the fundamental value, that is k_s and k_p, and for the average expectation, that is $\bar{k}_s$ and $\bar{k}_p$. The graphs show the variation in Kalman gains across moneyness for options with a fixed time-to-expiration of 12 months. For contracts with moneyness close that 100, dealers almost put full weight on their private signal both when updating their expectations about the current fundamental value (top left panel) and their expectation about the location of the current average expectation across dealers (bottom left panel). The consensus price signal receives correspondingly low weight in the updating of expectations about fundamental values (top right panel) and expectations about the location of average expectations (bottom right panel). As we move towards OTM contracts, both put and call options, the consensus price increases in importance and the weight on private signals decreases. Table 4 in the Appendix reports estimates for all contracts. They confirm the pattern observed for contracts with 12 months to expiration. The Kalman gains for the consensus price are higher for more extreme options contracts that are predominately OTC-traded. In market segments that overlap with exchange-based trading, dealers' private signals receive a higher weight in updating expectations and the consensus price is relatively less important. Lastly, note that for a given contract the Kalman gain for the consensus price is always higher for updating expectations about the location of the average expectation than for updating expectations about the fundamental value.

A key structural parameter for understanding this variation in Kalman gains across market segments is $1/\sigma_\eta$, the precision of the private signal. The estimates for σ_η, given in the fourth row of Table 2, show that dealers receive very precise private signals for contracts that overlap with active exchange-based trading activity.[19] Consequently, the implied Kalman gains for these contracts in Figure 3 show that dealers put essentially full weight on their private signal and largely ignore the information contained in the consensus price when updating expectations about θ_t. For option contracts with low exchange-based trading activity, the private signals are estimated to be noisier. Therefore, increasingly more weight is given to the consensus price. When updating expectations about $\bar{\theta}_t$, the consensus price receives relatively higher weight for all contracts. This highlights the scarcity of information in these market segments, but it also illustrates the strategic value of the consensus price as a focal public signal. Row five of Table 2 reports that 1-k, the weight dealers put on their prior expectation as given in (12), is estimated to be strictly positive for all contracts. Hence, dealers do not put full weight on new information, especially not for exclusively OTC-traded contracts. Furthermore, ρ_{12} is estimated to be strictly smaller than 1. This implies that a dealer's forecast errors for fundamental and average expectation are not perfectly correlated. As explained in Section 5.1, this leads to long-lived private information and, consequently, dispersed priors among dealers.

5.1.2 Uncertainty smile and term structure

As dealers' beliefs are normal distributions, we display valuation and strategic uncertainty in terms of 95% posterior intervals. That is, having observed past period's consensus price and the current private signal, a dealer attributes a probability of 0.95 to the event that the current fundamental value lies in an interval of length $3.92\,\sigma^p_{11}$ centered around its current expectation $\theta_{i,t}$. The length of this posterior interval thus gauges a dealer's uncertainty about the fundamental value θ_t at the time of submission. Similarly, for strategic uncertainty, the length of the posterior interval is $3.92\,\sigma^p_{22}$ and it is centred around $\bar{\theta}_{i,t}$. It measures a dealer's uncertainty about the location of the average expectation for θ_t across dealers.

[19]Table 2 provides the parameter estimates for the contracts with time-to-expiration of 12 months. In the Appendix we provide the estimates for the complete contract space. Table 3 provides estimates for ρ, σ_u, σ_ε and σ_η, and Table 5 for k and ρ_{12}.

Table 2: Sample parameter estimates and implied quantities

	60	80	90	95	100	105	110	120	150
ρ	0.967	0.930	0.939	0.949	0.941	0.930	0.949	0.967	0.969
	(0.015)	(0.024)	(0.027)	(0.028)	(0.022)	(0.025)	(0.022)	(0.013)	(0.017)
σ_u	0.047	0.076	0.082	0.086	0.091	0.095	0.095	0.073	0.135
	(0.001)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.002)	(0.003)
σ_ε	0.055	0.004	0.006	0.007	0.009	0.011	0.014	0.036	0.262
	(0.004)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)	(0.016)
σ_η	0.041	0.014	0.010	0.010	0.010	0.011	0.015	0.036	0.281
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)
k	0.733	0.998	0.996	0.996	0.995	0.993	0.989	0.901	0.490
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.010)	(0.014)
ρ_{12}	0.974	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.922
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)

This table presents the mean and standard deviation of the model parameter estimates and implied quantities for contracts with time-to-expiration of 12 months. The header gives the moneyness of contracts. Rows 1 to 6 show estimates for the persistence of the process for the fundamental, ρ, and the standard deviation of the shock to the fundamental, σ_u, the standard deviation of noise in consensus price, σ_ε, the standard deviation of noise of the private signal, σ_η, the weight dealers put on new information, k, and the correlation between the forecast error for fundametal value and average expectations, ρ_{12}. Estimates are obtained using MCMC methods assuming diffuse priors for all parameters. The standard deviation of the posterior distribution of the parameter is given in parentheses below its mean (0.000 signifies standard deviations below 0.0005). The sample period is December 2002 to February 2015.

Figure 4 shows these posterior intervals for valuation uncertainty (left panels) and strategic uncertainty (right panels). As dealers' expectations are varying over time, we center all posterior intervals around the time-series mean of the corresponding consensus price. For our purpose, this is immaterial. Given the stationarity of the model, the lengths of the posterior intervals do not vary over time. The top panels show the "uncertainty smile", that is posterior intervals for fixed times-to-expiration of 12 months (black) and 5 years (red) across moneyness. The bottom panels display the term structure of uncertainty, i.e. they show how posterior intervals for ATM puts with moneyness 100 (black) and OTM put with moneyness 60 (red) vary by times-to-expiration. Table 6 in the Appendix reports the length of 95% posterior intervals for all contracts. The two top panels in Figure 4 show the well-known "smile" of the implied volatility curve. OTM put options (moneyness below 100) tend to be relatively more expensive than ATM put options or OTM call options reflecting market participants' demand for insurance against drops in the S&P 500 index. The width of the posterior intervals shows that for options with more extreme strike prices (further away from moneyness 100), valuation and strategic uncertainty are higher. These areas correspond to

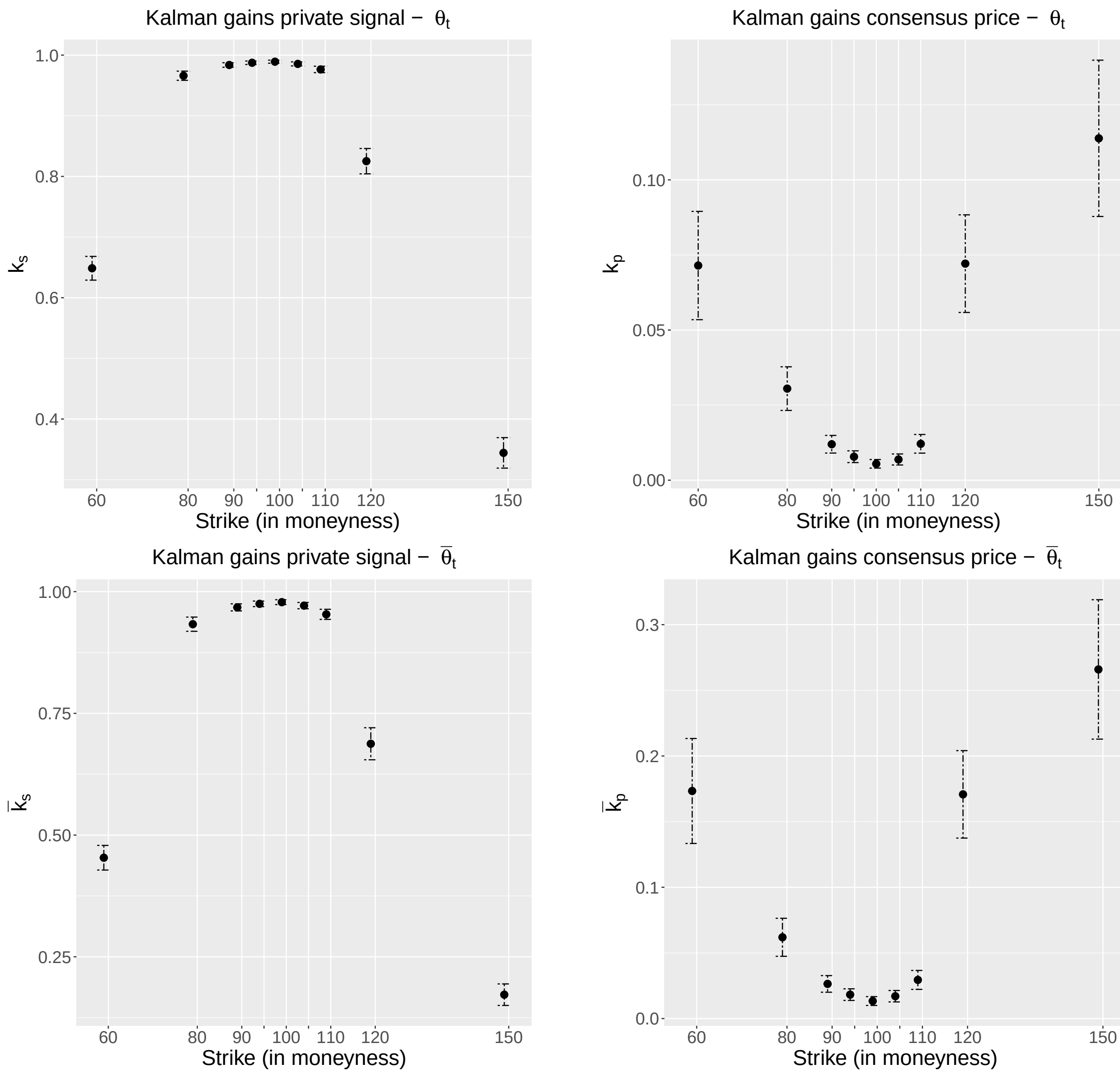


Figure 3: These figures present a dealer's Kalman gains for private signals and consensus prices. The horizontal axis denotes the moneyness of the option contracts. The black dots in the figures represent the Kalman gain extracted from the K matrix in Equation (9). The *top* figures depict k_s and k_p. From left to right, these are the weights put on the private signal and consensus price when updating the posterior expectation about the fundamental θ_t. The *bottom* figures depict $\bar{k}_s$ and $\bar{k}_p$. From left to right, these are the weights put on the private signal and consensus price when updating the posterior expectation about average expectations $\bar{\theta}_t$. The 95% centred interval of the posterior distribution of Kalman gain estimates are given by the dotted lines surrounding the dots. The Kalman gains are for the option contracts with a **time-to-expiration** of **12 months**. The sample period is December 2002 to February 2015.

market segments in which trading is predominantly or exclusively OTC, as was previously shown in Figure 1. For options with moneyness 150 and time-to-expiration of 12 months, for example, the posterior intervals are on the order of 8 volatility points. This is substantial given that the time-series average and standard deviation of the consensus price for this contract are 13 and 3.8 volatility points, respectively. While the term structure of uncertainty is relatively flat for ATM options (black posterior intervals in bottom panels), it is downward sloping for OTM puts (red posterior intervals in bottom panels).

These results reflect the estimated lower precision of the private signal, σ_η, for OTM contracts and especially OTM call options, as can be seen in row four of Table 2. Given the relatively low precision of private signals, more weight is put on prior expectations. This, in turn, is the source of "long-lived" private information and sizable strategic uncertainty. It contrasts to posterior intervals well below one volatility point for options in market segments with likely more trading activity. Here, private signals are precise, which implies lower values of σ^p_{11}. As all dealers are symmetrically informed and receive private signals from the same distribution, strategic uncertainty is small as well. This difference in results illustrates that for the exclusively OTC-traded areas of the option market, dealers are not only relatively uncertain about the correctness of their own option valuations, but also face substantial uncertainty about the relative position of their valuation to the average market valuation.

5.2 The informational properties of consensus prices

We now consider two counterfactual information structures for the options markets to gauge the informational content of the consensus price and its influence on dealer banks' uncertainty. We assume that the structural parameters of the model are invariant to these informational experiments. In particular, we assume that dealer banks do not adjust their information acquisition strategy, which would affect the precision of the private signal.[20] This assumption is not problematic when we use the counterfactual settings to measure the informational value of the consensus price in the current market setting. However, it becomes important when we consider the implications of changing the market's information structure.

To study the informativeness of the consensus price for dealer banks, we consider an information structure under which dealers only receive private signals. Denote by Σ^s the

[20]If private signals derive from OTC trading activity this would, for example, imply that dealers do not change their trading activity to generate more valuation information through experimentation.

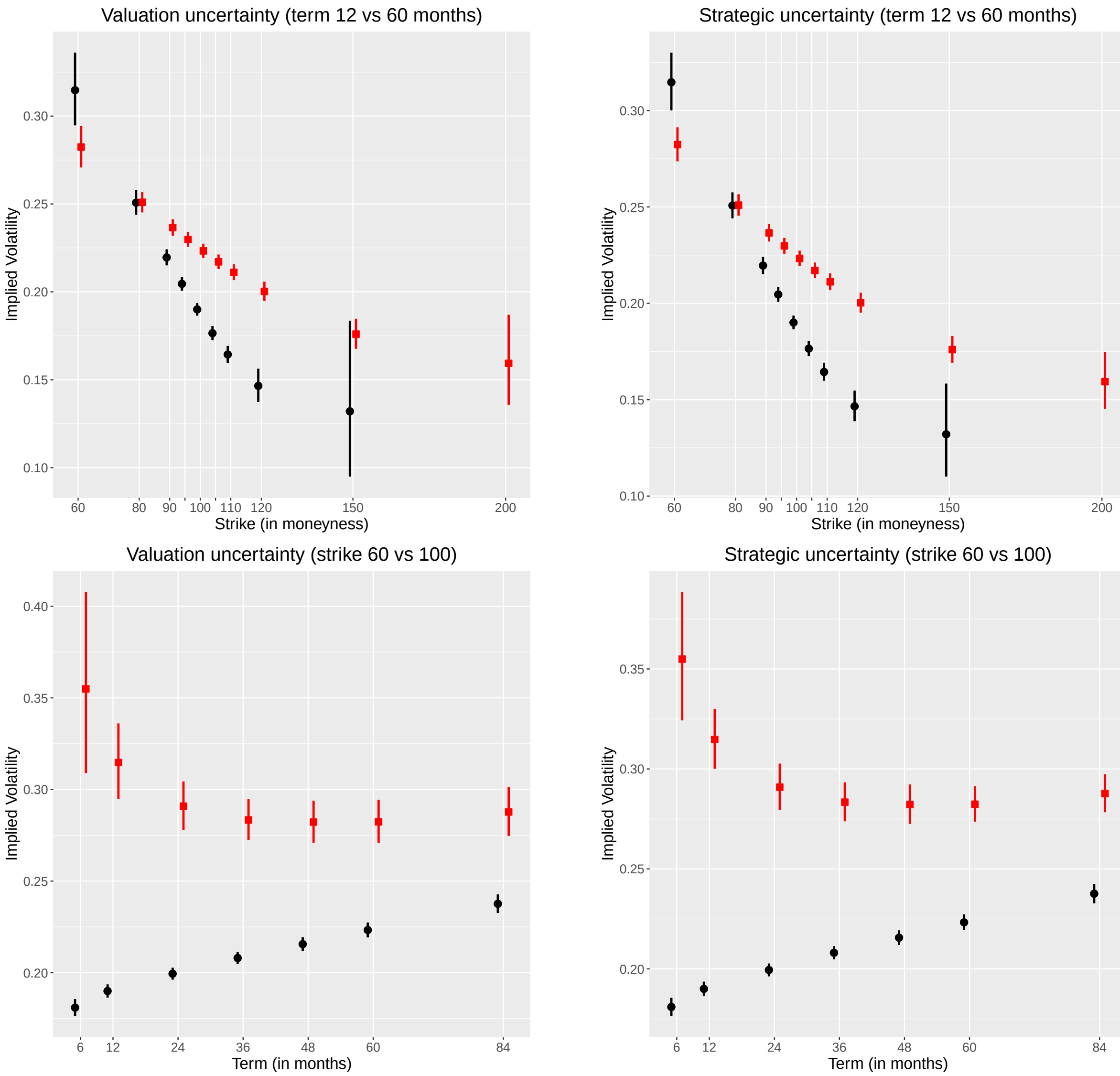


Figure 4: These figures display the variance of dealers' posterior beliefs expressed in terms of posterior intervals centred on the sample mean of the corresponding consensus price. The *left* figures depict the 95% posterior intervals for the fundamental value θ_t, $[p \pm 1.96 \cdot \sigma^p_{11}]$, as given in (9). The figures on the *right* display the posterior intervals for the average expectation $\bar{\theta}_t$, $[p \pm 1.96 \cdot \sigma^p_{22}]$. p is the time-series average of the corresponding consensus price. The two *top* panels depict the posterior intervals by moneyness for the option contracts with a fixed **time-to-expiration** of **12 months** and **60 months**. The two *bottom* panels show the term structure of the uncertainties for ATM options with fixed moneyness **100** and OTM options with fixed moneyness of **60**. The sample period is December 2002 to February 2015

covariance matrix of dealer i's posterior beliefs under this counterfactual information set, namely $\Omega^s_{i,t} = \{s_{i,t}, \Omega^s_{i,t-1}\}$. This covariance matrix can be obtained by solving a standard single-agent learning problem and evaluating it at our parameter estimates for $\{\rho, \sigma_u, \sigma_\eta\}$.[21] We use the percentage reduction in uncertainty that results from having access to the consensus price as a measure of price informativeness,

$$\Delta_i^p = \frac{(\sigma_{ii}^s - \sigma_{ii}^p)}{\sigma_{ii}^s}.$$

Note that we evaluate the informativeness of the consensus price both with respect to valuation information, that is information about θ_t, as given by Δ_1^p and with respect to strategic information, that is information about $\bar{\theta}_t$, as given by Δ_2^p.

To elicit the efficiency of the consensus price mechanism in aggregating dispersed information, we introduce a counterfactual setting with a fully efficient consensus price that perfectly reveals last period's fundamental value. As the price reveals θ_{t-1}, it is a common prior shared by all dealers before they receive new private signals in t. In addition to providing a benchmark for efficiency, this counterfactual also helps us understand how big an impediment the lack of a common prior is for creating a common understanding of market conditions. We denote by Σ^θ the counterfactual covariance matrix of posterior beliefs for a dealer who receives a fully efficient consensus price in the above sense. We measure the inefficiency of the consensus price by the increase in the standard deviation of posterior beliefs when moving from a fully efficient price to the current consensus price. We express this increase as a ratio to the standard deviation of posterior beliefs without consensus price,

$$\Delta_i^\theta = \frac{\left(\sigma_{ii}^p - \sigma_{ii}^\theta\right)}{\sigma_{ii}^s}.$$

We use this somewhat unusual definition of inefficiency to obtain the following decomposition,

$$1 = \underbrace{\frac{(\sigma_{ii}^s - \sigma_{ii}^p)}{\sigma_{ii}^s}}_{\substack{\Delta_i^p \\ \text{Price informativeness}}} + \underbrace{\frac{\left(\sigma_{ii}^p - \sigma_{ii}^\theta\right)}{\sigma_{ii}^s}}_{\substack{\Delta_i^\theta \\ \text{Price inefficiency}}} + \underbrace{\frac{\sigma_{ii}^\theta}{\sigma_{ii}^s}}_{\substack{\Delta_i^R \\ \text{Residual informational} \\ \text{friction}}} \tag{13}$$

[21] In Appendix 7.4 we derive the stationary posterior covariance matrices for first- and second-order beliefs for all counterfactual informational scenarios.

Given the lagged nature of the consensus pricing mechanism, even a fully efficient price does not eliminate all uncertainty about asset values. We also quantify the potential for further uncertainty reduction that is outside the scope of this specific information aggregation mechanism and refer to it as the residual informational friction. The potential for uncertainty reduction in a market structure without consensus prices can thus be decomposed into (consensus) price informativeness, (consensus) price inefficiency, and residual informational frictions.

The influence of information structure on uncertainty

Figure 5 displays the percentage reductions in uncertainty under the different informational settings for contracts with a fixed time-to-expiration of 12 months. The dark gray regions show price informativeness measures Δ_1^p and Δ_2^p across contracts, that is the percentage reduction in valuation, respectively, strategic uncertainty that results from having access to the consensus price. The lack of uncertainty reduction in the moneyness range from 80 to 110 is to be expected as dealers receive very precise private signal for these contracts. This mirrors the previously shown estimates of the Kalman gain k_s and $\bar{k}_s$ which are both close to 1 (see Figure 3). In the mostly OTC-traded market segments Δ_1^p is higher, which implies that the consensus price is more informative about θ_t. Here, the Kalman gains for the consensus price, k_p, are also higher. Table 7 in the Appendix shows similar patterns for other times-to-expiration. For all contracts under consideration, the reduction in valuation uncertainty is between 0% for the ATM contracts to 4.36% for the more extreme contracts. The comparison between Δ_1^p and Δ_2^p, that is the left and right panel in Figure 5, shows that the consensus price signal is much more informative about $\bar{\theta}_t$. Δ_2^p, the reduction in strategic uncertainty that results from having access to the consensus price, ranges from 0.02% to 27.33% (see Table 7). The relative larger decrease in strategic uncertainty in comparison to valuation uncertainty points to the importance of the consensus price for learning about strategic aspects of the market. This is also echoed in the difference between k_p and $\bar{k}_p$. Given the scarcity of shared trade data in market segments that are dominated by OTC trading, the ability of the consensus price to significantly reduce strategic uncertainty is both intuitive and important.

The light gray areas in Figure 5 display price inefficiency measures Δ_1^θ and Δ_2^θ, that is the additional percentage reductions in valuation, respectively, strategic uncertainty that could be achieved by moving from the current consensus price to a consensus price that perfectly reveals θ_{t-1}. Knowing the previous period's option value eliminates two sources of uncertainty. First, it eliminates the uncertainty that originates from the additive noise component of the consensus price, ε_t. Second, dealers no longer have to take into account the dispersion in prior expectations across dealers. With a perfect consensus price, they all start from a common prior before observing their new private signals. In the top and bottom panel of Figure 5, the lack of uncertainty reduction in the moneyness range from 80 to 110 is mainly due to the high precision of private signals, as was the case for Δ_1^p and Δ_2^p. The signal-to-noise ratio σ_u/σ_η puts an upper bound on the weight the consensus price can receive when updating expectations. The consensus price can at most reveal the past value, while the private signal is a signal about the current value. This limits the potential impact of a fully efficient price on valuation uncertainty. For contracts with intermediate moneyness, little weight is put on prior expectations, thus limiting the potential of a fully efficient price to reduce uncertainty by providing a common prior. For contracts with extreme moneyness, the relative imprecision of the private signal shifts weight towards the consensus price and the prior. This explains the up to 33.46% drop in valuation uncertainty and 62.69% drop in strategic uncertainty for the deep OTM call options, as seen in Table 7 in the Appendix. A focal point, such as the commonly observed consensus price, helps to reduce dispersion in priors among dealers. It thereby lowers strategic uncertainty and fosters a common understanding of market conditions.

The white areas in the figures show the residual informational frictions Δ_1^R and Δ_2^R, that is the potential percentage reduction in valuation, respectively, strategic uncertainty that is outside of the scope of this consensus pricing mechanism. The reduced size of this area for more extreme contracts illustrates the importance of public information in the opaque parts of the market, especially in providing information about other dealers' valuations. For contracts with moneyness between 80 and 110, informational frictions that could not be remedied by a perfectly efficient consensus pricing mechanism dominate. Reducing the remaining uncertainty would require changing the design parameters of the consensus pricing service. Increasing the frequency of the consensus service, for example, can be thought of as lowering the variance of the fundamental shocks, σ_u^2, in our model. However, given the

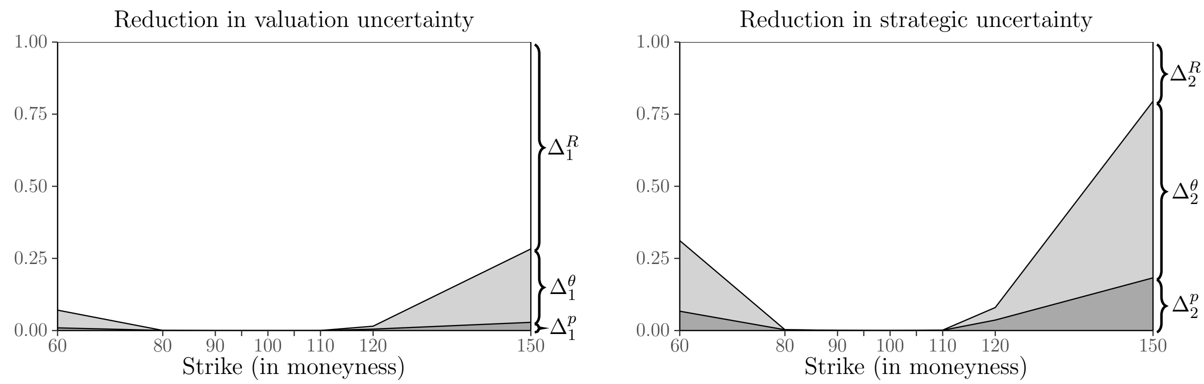


Figure 5: These two figures present the percentage reductions in valuation and strategic uncertainty under different informational settings. The left figure shows the percentage reduction in **valuation uncertainty** (y-axis) and the right figure shows the percentage reductions in **strategic uncertainty** (y-axis) by moneyness (x-axis) for the option contracts with a **time-to-expiration** of **12 months**. In the base case setting, dealers only observe private signals. This is indicated by the lower horizontal axis. The dark grey area is the percentage reduction in uncertainty due to observing the consensus price, i.e., Δ_i^p in (13). The light grey area indicates the further reduction in uncertainty due to observing the past state, i.e., Δ_i^θ. The white area is the further reduction in uncertainty that can be achieved from an information structure that eliminates informational frictions, i.e., Δ_i^R. The sample period is from December 2002 to February 2015.

labour-intensive nature of the consensus pricing process, running a more frequent service is costly. It appears that the marginal cost of increasing the frequency of the service exceeds the dealers' willingness to pay for a marginal reduction in uncertainty.

6 Conclusion

In this paper we provide empirical evidence on the ability of consensus prices to reduce valuation and strategic uncertainty among major dealer banks in the over-the-counter options market. This evidence is based on a structural model of learning from prices. The estimation uses a proprietary panel of price estimates that large broker-dealers have provided to a consensus pricing service for OTC derivatives. The structural model allows us to address three questions. First, how uncertain are dealer banks that participate in the OTC derivatives market about the values of S&P 500 index options? Here we consider two dimensions of uncertainty: a dealer bank's uncertainty about fundamental values and its uncertainty about its relative position with respect to other market participants' valuations. Second, does the consensus price feedback help to reduce market participants' uncertainty? Last, how well does the consensus pricing mechanism aggregate dispersed information?

We find that both valuation and strategic valuation uncertainty vary substantially across the different market segments. Dealers are more uncertain about option valuations for contracts that are predominately or exclusively traded in the OTC segment of the options market. For these contracts, they are also less certain about their relative position in relation to other market participants. Dealer banks do not appear to rely heavily on the consensus price feedback to reduce valuation uncertainty. The consensus price feedback is found to be most important for reducing strategic uncertainty, and particularly so for options with extreme contract terms. This result is consistent with the scarcity of shared valuation information for such extreme contracts. It stresses the importance of publicly observable valuation data, such as benchmarks, to establish a shared understanding of market conditions in OTC markets. Such a shared understanding can be particularly valuable during episodes of market stress where high levels of strategic uncertainty can cause financial markets to freeze.

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7 Appendix

7.1 Solution algorithm

Here, we show how to solve the consensus pricing problem using a solution algorithm developed in Nimark (2017). We adopt the following standard notation for higher-order expectations, defining recursively

$$\theta_t^{(0)} = \theta_t,$$
$$\theta_{i,t}^{(k+1)} = \mathbb{E}\left(\theta_t^{(k)} | \Omega_{i,t}\right) \quad \text{and} \quad \theta_t^{(k+1)} = \int_0^1 \theta_{i,t}^{(k+1)} \, di \text{ for all } k \geq 0.$$

We denote institution i's hierarchy of expectations up to order k by

$$\theta_{i,t}^{(1:k)} = \left(\theta_{i,t}^{(1)}, ..., \theta_{i,t}^{(k)}\right)^{\mathsf{T}}$$

and for the hierarchy of average expectations up to order k, including the fundamental value $\theta_t^{(0)}$ as first element,

$$\theta_t^{(0:k)} = \left(\theta_t^{(0)}, \theta_t^{(1)}, ..., \theta_t^{(k)}\right)^{\mathsf{T}}.$$

The solution procedure proceeds recursively. It starts with a fixed order of expectations $k \geq 0$ and postulates that the dynamics of average expectations $\theta_t^{(0:k)}$ are given by the VAR(1)

$$\theta_t^{(0:k)} = M_k \, \theta_{t-1}^{(0:k)} + N_k \, w_t, \tag{14}$$

with $w_t = (u_t, \varepsilon_{t-1})^{\mathsf{T}}$ and $\theta_t^{(n)} = \theta_t^{(k)}$ for all $n \geq k$.

Institution i's signal can be expressed in terms of current and past average expectations, $\theta_t^{(0:k)}$ and $\theta_{t-1}^{(0:k)}$, and the period t shocks w_t and $\eta_{i,t}$. The private signal can be written as

$$s_{i,t} = e_1^{\mathsf{T}} \, \theta_t^{(0:k)} + \sigma_\eta \, \eta_{i,t},$$

where e_j denotes a column vector of conformable length with a 1 in position j, all other elements being 0. Similarly, we can express the consensus price p_t as

$$p_t = \theta_t^{(1)} + \sigma_\varepsilon \, \varepsilon_t = e_2^\top \, \theta_t^{(0:k)} + \sigma_\varepsilon \, \varepsilon_t.$$

Denote the vector of signals by $z_{i,t} = (s_{i,t}, p_{t-1})^\top$. We can now express the signals in terms of current and past average expectations and shocks,

$$z_{i,t} = D_{k,1} \, \theta_t^{(0:k)} + D_{k,2} \, \theta_{t-1}^{(0:k)} + R_w \, w_t + R_\eta \, \eta_{i,t}, \tag{15}$$

where

$$D_{k,1} = \begin{bmatrix} e_1^\top \\ 0_{k+1}^\top \end{bmatrix}, \quad D_{k,2} = \begin{bmatrix} 0_{k+1}^\top \\ e_2^\top \end{bmatrix}, \quad R_\eta = \begin{bmatrix} \sigma_\eta \\ 0 \end{bmatrix} \quad \text{and} \quad R_w = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_\varepsilon \end{bmatrix}.$$

We thus obtain a state space representation of the system from the perspective of institution i. Equation (14) describes the dynamics of the latent state variable $\theta_t^{(0:k)}$; Equation (15) is the observation equation that provides the link between the state and i's signals. Using a Kalman filter that allows for lagged state variables (Nimark 2015) allows us to express institution i's expectations conditional on the information contained in $\Omega_{i,t}$ as

$$\theta_{i,t}^{(1:k+1)} = M_k \, \theta_{i,t-1}^{(1:k+1)} + K_k \left[z_{i,t} - D_{1,k} \, M_k \, \theta_{i,t-1}^{(1:k+1)} - D_{2,k} \, \theta_{i,t-1}^{(1:k+1)} \right], \tag{16}$$

where K_k is the (stationary) Kalman gain. Substituting out the signal vector in terms of state variables and shocks, this can equivalently be written as

$$\begin{aligned} \theta_{i,t}^{(1:k+1)} = {} & [M_k - K_k(D_{1,k}M_k + D_{2,k})] \, \theta_{i,t-1}^{(1:k+1)} \\ & + K_k(D_{1,k}M_k + D_{2,k})\theta_{t-1}^{(0:k)} + K_k(D_{1,k}N_k + R_w)w_t + K_k R_\eta \, \eta_{i,t}. \end{aligned}$$

Averaging this expression across all submitters, assuming that by a law of large numbers $\int_0^1 \eta_{i,t} \, di = 0$, average expectations are then given by

$$\begin{aligned} \theta_t^{(1:k+1)} = {} & [M_k - K_k(D_{1,k}M_k + D_{2,k})] \, \theta_{t-1}^{(1:k+1)} \\ & + K_k(D_{1,k}M_k + D_{2,k})\theta_{t-1}^{(0:k)} + K_k(D_{1,k}N_k + R_w)w_t. \end{aligned}$$

Combined with the fact that $\theta_t^{(0)} = \rho\,\theta_{t-1}^{(0)} + \sigma_u\,u_t$, we now obtain a new law of motion for the state,

$$\theta_t^{(0:k+1)} = M_{k+1}\,\theta_{t-1}^{(0:k+1)} + N_{k+1}\,w_t,$$

with

$$M_{k+1} = \begin{bmatrix} \rho\,e_1^\intercal & 0 \\ K_k(D_{1,k}M_k + D_{2,k}) & 0_{k\times 1} \end{bmatrix} + \begin{bmatrix} 0 & 0_{1\times k} \\ 0_{k\times 1} & M_k - K_k(D_{1,k}M_k + D_{2,k}) \end{bmatrix} \tag{17}$$

and

$$N_{k+1} = \begin{bmatrix} \sigma_u\,e_1^\intercal \\ K_k(D_{1,k}N_k + R_w) \end{bmatrix}. \tag{18}$$

Note, however, that now the state space has increased by one dimension from $k+1$ to $k+2$. This is a consequence of the well-known infinite regress problem when filtering endogenous signals. When filtering signals based on average expectations of order k, institutions have to form beliefs about average expectations of order k. But this implies that equilibrium dynamics are influenced by average expectations of order $k+1$, and so on for all orders $k \geq 0$.

In practice, the solution algorithm works as follows. We initialize the iteration at $k=0$ with $M_0 = \rho$ and $N_0 = \sigma_u$, which implies that $\theta_t^{(1)} = \theta_t^{(0)}$ for all t. Consequently, the consensus price of the first iteration is given by[22]

$$p_t^{[1]} = \theta_t^{(0)} + \sigma_\varepsilon\,\varepsilon_t.$$

This yields a Kalman gain K_0 (here a two-dimensional vector) which can then be used to obtain M_1 and N_1 via equations (17) and (18) and so on until either convergence of the process $p_t^{[n]}$ has been achieved according to a prespecified convergence criteria after n steps or a upper bound on steps has been reached. The highest-order belief that is not trivially defined by lower-order beliefs is then of order n.

[22]Superscripts in square brackets denote iterations of the algorithm.

7.2 Kalman Filter for Estimation

For a given contract, that is, a given time-to-expiration, moneyness, and option type (put or call), our data consists of two time series. Let S be the total number of institutions that have submitted to Totem over the course of our sample and let $\iota_t \subset \{1, 2, .., S\}$ be the set of institutions active in t.[23] Our sample of submissions is then given by $(\mathbf{p}_t)_{t=1}^T$, where $\mathbf{p}_t = (p_{j,t})_{j\in\iota_t}$ is a $|\iota_t|$-dimensional vector consisting of the individual period t consensus price submissions. We assume that consensus price submissions are institution i's conditional expectation of θ_t,

$$p_{i,t} = \theta_{i,t}. \tag{19}$$

Following our model, we assume that the consensus price of period t, p_t, equals the average expectation of period t plus aggregate noise, that is,

$$p_t = \bar{\theta}_t + \sigma_\varepsilon\, \varepsilon_t.$$

Our data set for a given contract, $(\mathbf{y}_t)_{t=1}^T$, then consists of the time-series of institutions' price submissions for this contract and the corresponding consensus price of the previous period, i.e. $\mathbf{y}_t = (p_{t-1}\,, \mathbf{p}_t)^\mathsf{T}$.[24]

To estimate the model, we fix the maximum order of beliefs at $\bar{k} = 4$.[25] Average expectations then evolve according to (5), namely,

$$x_t = M_{\bar{k}}\, x_{t-1} + N_{\bar{k}}\, v_t,$$

where x_t is the $\bar{k}+1$ dimensional state vector, $M_{\bar{k}}$ and $N_{\bar{k}}$ are functions of the parameters ϕ defined recursively (see Appendix 7.1 for solution algorithm) and $v_t = (u_t\,, \varepsilon_{t-1})^\mathsf{T} \sim N(\mathbf{0}_2, I_2)$.[26] The dynamics of institution i's conditional expectations $x_{i,t}$ can be expressed in

[23] If an institution does not submit a price in t, we treat this as a missing value. However, it is assumed that this institution received both the consensus price and the private signal about the fundamental in that period.

[24] This timing convention of lagging the consensus price simplifies the expression for the likelihood function in terms of $\mathbf{y}_t$.

[25] Allowing $\bar{k}$ greater than 4 does not change the estimates noticeably.

[26] We use $0_{n\times m}$ to denote a $n \times m$ matrix of zeros, 1_n is a (column) vector containing n ones, and I_n is an n-dimensional identity matrix.

terms of deviations from average expectations, $\hat{x}_{i,t} \equiv x_{i,t}(1:\bar{k}) - x_t(2:(\bar{k}+1))$,[27]

$$\hat{x}_{i,t} = Q_{\bar{k}}\,\hat{x}_{i,t-1} + V_{\bar{k}}\,\eta_{i,t},$$

where

$$Q_{\bar{k}} = \left[M_{\bar{k}} - K_{\bar{k}}(D_{1,\bar{k}}M_{\bar{k}} + D_{2,\bar{k}})\right] \text{ and } V_{\bar{k}} = K_{\bar{k}}\,R_\eta.$$

Given the linearity of the above system and the assumed normality of shocks, the likelihood function for the observed data $(\mathbf{y})_{t=1}^{T}$ with $\mathbf{y}_t = (p_{t-1}\,, \mathbf{p}_t)^\top$ can be derived using the Kalman filter. We define $\alpha_t = (x_t^\top\,, x_{1,t}^\top\,, ..., x_{S,t}^\top\,, \varepsilon_{t-1})^\top$ to be the state of the system in t.

The *transition equation* of the system in state space form is then given by

$$\alpha_t = T\alpha_{t-1} + R\,\epsilon_t,$$

where

$$T = \begin{pmatrix} M_{\bar{k}}\,,\ 0_{\bar{k}+1\times S\bar{k}+1} \\ 0_{S\,\bar{k}\times\bar{k}+1}\,,\ I_S \otimes Q_{\bar{k}}\,,\ 0_{S\,\bar{k}\times 1} \\ 0_{2\times\bar{k}+1+S\bar{k}+1} \end{pmatrix}\ ,\ R = \begin{pmatrix} N_{\bar{k}}\,,\ 0_{\bar{k}+1\times S} \\ 0_{S\bar{k}\times 2}\,,\ I_S \otimes \sigma_\eta\,V_{\bar{k}} \\ I_2\,,\ 0_{2\times S} \end{pmatrix}$$

and $\epsilon_t = (u_t\,, \varepsilon_{t-1}\,, \eta_{1,t}\,, ..., \eta_{S,t})^\top \sim N\left(\mathbf{0}_{2+S}, I_{2+S}\right)$.

We now derive the *observation equation* for the system. First note that the consensus price p_{t-1} can be expressed in terms of the past state vector α_{t-1} as

$$p_{t-1} = \bar{\theta}_{t-1} + \sigma_\varepsilon\,\varepsilon_{t-1} = e_2^\top\,\alpha_{t-1} + \sigma_\varepsilon\,e_{\text{last}}^\top\,\alpha_t.$$

Next, note that we can write institution i's submission $p_{i,t}$ as

$$p_{i,t} = \theta_{i,t} = e_2^\top\,\alpha_t + e_n^\top\,\alpha_t,$$

where n corresponds to the position of the first element of $x_{i,t}$ in α_t. It follows that our observations of the system, $\mathbf{y}_t$, can be expressed in terms of the latent state of the system,

[27] $x(j:k)$ denotes a subvector of x containing elements j to k.

namely

$$\mathbf{y}_t = Z_{1,t}\,\alpha_t + Z_{2,t}\,\alpha_{t-1}.$$

$Z_{1,t}$, and $Z_{2,t}$ are matrices that depend on the parameters of the model. To derive these matrices, we start by defining an auxiliary matrix J_t that allows us to deal with missing submissions by some institutions in period t. Recall that $\iota_t \subset \{1, 2, .., S\}$ is the set of institutions submitting in t. Let $\iota_{k,t}$ designate the k-th element of the index ι_t. J_t is a $(|\iota_t| \times S)$ matrix whose k-th row has a 1 in position $\iota_{k,t}$ and zeros otherwise. We have $Z_{1,t} = J_t\,Z_1$ and $Z_{2,t} = J_t\,Z_2$, where

$$Z_1 = \begin{pmatrix} 0_{1\times 1+\bar{k}+S\bar{k}}\,,\,\sigma_\varepsilon \\ 0\,,\,1\,,\,e_1^\intercal \\ 0\,,\,1\,,\,e_{\bar{k}+1}^\intercal \\ \vdots \\ 0\,,\,1\,,\,e_{(S-1)\bar{k}+1}^\intercal \end{pmatrix}, \quad \text{and} \quad Z_2 = \begin{pmatrix} 0, 1, 0_{1\times(\bar{k}-2)+S\bar{k}+1} \\ 0_{1\times 1+\bar{k}+S\bar{k}+1} \\ \vdots \\ 0_{1\times 1+\bar{k}+S\bar{k}+1} \end{pmatrix}.$$

Given a prior for the state of the system at $t = 1$, $\alpha_1 \sim N(\mathbf{a}_1, P_1)$, we can now apply the usual Kalman filter recursion to derive the likelihood function for our data $(\mathbf{y}_t)_{t=1}^T$ given the parameter vector ϕ denoted $L\left((\mathbf{y}_t)_{t=1}^T \mid \phi\right)$.

7.3 Proof of identification

Strategy of proof The proof of identification proceeds in two steps. First, we establish identification for the model under the assumption that submitting institutions take the consensus price to be an exogenous signal of the current state, i.e. $p_t = \theta_t + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. This is the model of the first step in Nimark (2017)'s solution algorithm. Second, once we have established identification of the first-step model, we proceed by induction. In particular, we argue that if the model is identified at step n of the algorithm, it is also identified at step $n + 1$. This then establishes identification of the model at all steps of the algorithm.

A. Identification with exogenous consensus price signal

If submitters assume that the consensus price is an exogenous signal of the (past) state, then individual submitters' first-order beliefs are updated according to

$$\theta_{i,t} = \rho\,\theta_{i,t-1} + (k_{11}\ k_{12}) \begin{pmatrix} \theta_t + \eta_{i,t} - \rho\,\theta_{i,t-1} \\ \theta_{t-1} + \varepsilon_{t-1} - \theta_{i,t-1} \end{pmatrix},$$

where $\eta_{i,t} \sim N(0, \sigma_\eta^2)$. We can write this as

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + k_{12}\,\varepsilon_{t-1} + k_{11}\,\eta_{i,t}, \tag{20}$$

where the Kalman gains k_{11} and k_{12} are given by

$$k_{11} = \frac{\zeta + \rho^2\,k}{\zeta + \rho^2 + \psi/(1-\psi)} \text{ and } k_{12} = \rho(k - k_{11}) \text{ with}$$

$$k = \frac{1}{2} + \frac{1}{2\rho^2}\left\{\left[(1-\rho)^2 + \xi\right]^{\frac{1}{2}}\left[(1+\rho)^2 + \xi\right]^{\frac{1}{2}} - (1+\xi)\right\},$$

$$\xi = \frac{\zeta}{\psi}\ ,\ \psi = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \text{ and } \zeta = \frac{\sigma_u^2}{\sigma_\varepsilon^2}.$$

The average first-order belief is then

$$\bar\theta_t = (1-k)\rho\,\bar\theta_{t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + k_{12}\,\varepsilon_{t-1},$$

with corresponding (step 2) consensus price process

$$p_t = \bar\theta_t + \varepsilon_t.$$

This implies the following dynamics for the consensus price,

$$p_t = (1-k)\rho\,p_{t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + (k_{12} - (1-k)\rho)\varepsilon_{t-1} + \varepsilon_t. \tag{21}$$

Observed data We assume that our observed data consists of a panel of individual first-order beliefs for S submitting institutions $\{\{\theta_{i,t}\}_{i=1}^S\}_{t=1}^T$ that evolve according to (20), and the corresponding time-series of consensus prices $\{p_t\}_{t=1}^{T}$ is generated by the process specified in (21).

We now show how the distribution of the above data identifies the model parameters of interest, namely $\{\rho, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_u^2\}$.

1. Deviations of the consensus price from average expectations identify σ_ε^2.
We obtain estimates for the error ε_t from the difference between the current consensus price and the current mean across submissions,

$$\varepsilon_t = p_t - \bar{\theta}_t.$$

We can thus identify σ_ε^2 from the time-series variance of the estimated errors.

2. Individual deviations from average expectations identify $(1-k)\rho$.
Individual deviations from the consensus, $\hat{\theta}_{i,t} = \theta_{i,t} - \bar{\theta}_t$ are given by

$$\hat{\theta}_{i,t} = (1-k)\rho\,\hat{\theta}_{i,t-1} + k_{11}\,\eta_{i,t}.$$

Individual deviations follow an AR(1) process. Deviations from consensus mean-revert more quickly if submitters put less weight on past information (higher k), or if the fundamental value process is less persistent (low ρ). We can therefore identify $(1-k)\rho$ from the auto-covariances of individual deviations from the current mean submission.

3. Persistence in consensus price updates identifies ρ and hence k via $(1-k)\rho$.
Having identified $(1-k)\rho$ we can obtain $\omega_t = p_t - (1-k)\rho\,p_{t-1}$ from our data, where

$$\omega_t = k_{11}\,u_t + k\,\rho\left(\frac{u_{t-1}}{1-\rho L}\right) + (k_{12} - (1-k)\rho)\varepsilon_{t-1} + \varepsilon_t.$$

ω_t is a noisy measure of the news about the fundamental value submitters receive in period t. By subtracting $(1-k)\rho\,p_{t-1}$ from p_t it "eliminates" their prior beliefs. For sufficiently long lags, ω_t's auto-correlation exclusively comes from its dependence on the fundamental process and not the aggregate noise, ε_t. Its auto-covariances thus allow us to identify the persistence in the process of θ_t. In particular, we can see that the auto-covariances of ω_t have to satisfy

$$Cov(\omega_t, \omega_{t-3}) = \rho\, Cov(\omega_t, \omega_{t-2}).$$

The ratio of these auto-covariances thus identify ρ,

$$\rho = Cov(\omega_t, \omega_{t-3})/Cov(\omega_t, \omega_{t-2}),$$

which together with $(1-k)\rho$ then allow us to identify $1-k$, i.e. the persistence in individual expectations due to informational frictions.

4. The weight submitters put on the consensus price when updating expectations identifies σ_η^2 and hence σ_u^2 via k.
k determines how much weight submitters put on new information as opposed to their priors. It is given by

$$k = \frac{1}{2} + \frac{1}{2\rho^2}\left\{\left[(1-\rho)^2 + \xi\right]^{\frac{1}{2}}\left[(1+\rho)^2 + \xi\right]^{\frac{1}{2}} - (1+\xi)\right\},$$

$$\text{where } \xi = \frac{\zeta}{\psi} \text{ with } \psi = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \text{ and } \zeta = \frac{\sigma_u^2}{\sigma_\varepsilon^2}.$$

It is a function of ξ, which is a ratio of the variance of the shocks to the fundamental value to the variance of the signal noises and can thus be seen as a measure of the signal to noise ratio. k is monotonically increasing in ξ; a higher signal to noise ratio implies a higher weight on current signals. Hence, having already identified k, we can also identify ξ.

In turn, the weights submitters put on the private signal and the consensus price can be expressed in terms of k, ξ, and ψ, namely

$$k_{11} = \frac{\xi\,\psi + \rho^2\,k}{\xi\,\psi + \rho^2 + \psi/(1-\psi)} \text{ and } k_{12} = \rho(k - k_{11}).$$

It can be shown that, for a given k, the weight on the private signal, k_{11}, is monotonically decreasing and the weight on the consensus price, k_{12}, monotonically increasing in ψ for $\psi \in (0,1)$; a relatively more noisy private signal will lead submitters to shift weight from the private signal to the consensus price (given k). As we have already identified k and ξ, knowing either k_{11} or k_{12} will allow us to identify ψ. Given ψ we can then back out σ_η^2 and ζ, which yields σ_u^2.

We now proceed to show identification of k_{12}, which by the previous argument establishes identification of the model. To do so, we return to the individual expectation updating equation,

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k_{11}\,\rho\,\theta_{t-1} + k_{12}\,p_{t-1} + k_{11}\,\eta_{i,t} + k_{11}\,u_t.$$

We also have

$$\theta_{i,t-1} = (1-k)\rho\,\theta_{i,t-2} + k_{11}\,\theta_{t-1} + k_{12}\,p_{t-2} + k_{11}\,\eta_{i,t-1}.$$

Multiplying the latter expression by ρ and subtracting from the former eliminates the unobservable θ_{t-1}. We obtain an expression in terms of observables and shocks,

$$\theta_{i,t} - \rho\,\theta_{i,t-1} = (1-k)\rho(\theta_{i,t-1} - \rho\,\theta_{i,t-2}) + k_{12}\,(p_{t-1} - \rho\,p_{t-2}) + k_{11}(\eta_{i,t} - \rho\,\eta_{i,t-1}) + k_{11}\,u_t.$$

Note that we have already identified $(1-k)\rho$. Define

$$y_{i,t} = \theta_{i,t} - \rho\,\theta_{i,t-1} - (1-k)\rho(\theta_{i,t-1} - \rho\,\theta_{i,t-2}).$$

We can then identify the coefficient k_{12} from the covariance of $y_{i,t}$ and $p_{t-1} - \rho\,p_{t-2}$ noting that

$$y_{i,t} = k_{12}\,(p_{t-1} - \rho\,p_{t-2}) + k_{11}(\eta_{i,t} - \rho\,\eta_{i,t-1}) + k_{11}\,u_t.$$

This is possible as p_{t-1} is a signal based on period $t-1$ information. It is not correlated with the shock u_t. Furthermore, the idiosyncratic noise terms $\eta_{i,t}$ and $\eta_{i,t-1}$ are uncorrelated with the consensus price process by construction.

B. Establishing identification by induction

Suppose we have established identification of the model parameters by our observed data for step n of the algorithm. That is, any two distinct sets of parameters ϕ_1 and ϕ_2 imply distinct distributions of the observable data. In particular, the step n consensus price process that submitters will assume in step $n+1$ differs across the two parameter sets. This necessarily implies that the distribution of individual expectations will differ across the two parameter sets in step $n+1$. This then establishes identification of the model at step $n+1$ of the algorithm.

7.4 Covariance matrices for counterfactuals

Consensus price perfectly reveals past state

If the consensus price perfectly aggregates dispersed information, we have

$$p_t = \theta_t.$$

In this case all submitters start period t with a common prior about θ_t, namely $\rho\,\theta_{t-1}$, and there is no higher-order uncertainty before receiving new signals. This is because every submitter knows that every submitter knows (and so on ...) that the average expected value of θ_t before receiving period t signals is $\rho\,\theta_{t-1}$.

Submitter i's expectations about the fundamental given signal $s_{i,t} = \theta_t + \eta_{i,t}$ can be obtained by the standard updating formula as state θ_t and signal $s_{i,t}$ given θ_{t-1} are jointly normally distributed:

$$\mathbb{E}_{i,t}\left(\theta_t\right) = \theta_{i,t} = \rho\,\theta_{t-1} + k_1\left(s_{i,t} - \rho\,\theta_{t-1}\right) = \rho\,\theta_{t-1} + k_1(u_t + \eta_{i,t})\,,$$

where k_1 is the Kalman gain

$$k_1 = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\eta^2}.$$

It follows that the average expectation is

$$\bar{\theta}_t = \rho\,\theta_{t-1} + k_1\,u_t.$$

Now define the random vector

$$X_t = \left[\theta_t - \rho\,\theta_{t-1}\,,\, \bar{\theta}_t - \rho\,\theta_{t-1}\right] = \left[u_t\,,\, k_1\,u_t\right]$$

and

$$y_{i,t} = s_{i,t} - \rho\,\theta_{t-1} = u_t + \eta_{i,t}.$$

X_t and $y_{i,t}$ are jointly normally distributed. Thus, the covariance of X_t given $y_{i,t}$ is

$$Var\left(X_t | y_{i,t}\right) = \Sigma_{xx} - \Sigma_{xy}\left(\sigma_y^2\right)^{-1}\Sigma_{xy}^{\mathsf{T}},$$

where Σ_{xx} is the variance of X_t and Σ_{xy} is the covariance of X_t and $y_{i,t}$, namely,

$$\Sigma_{xx} = \begin{bmatrix} \sigma_u^2 & k_1\sigma_u^2 \\ k_1\sigma_u^2 & k_1^2\,\sigma_u^2, \end{bmatrix} \;,\; \Sigma_{xy} = \left[\sigma_u^2\;,\; k_1\,\sigma_u^2\right]^{\mathsf{T}}.$$

As $\rho\,\theta_{t-1}$ is known in t, $Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\Omega_{i,t}) = Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\theta_{t-1}, y_{i,t}) = Var\left(X_t|y_{i,t}\right)$. It follows that

$$Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\Omega_{i,t}) = \begin{bmatrix} \frac{\sigma_u^2\,\sigma_\eta^2}{\sigma_u^2+\sigma_\eta^2} & \frac{\sigma_u^4\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^2} \\ \frac{\sigma_u^4\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^2} & \frac{\sigma_u^6\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^3} \end{bmatrix}.$$

No consensus price feedback

Without consensus price feedback, the stationary expectation dynamics of submitter i are given by

$$\theta_{i,t} = \rho\,\theta_{i,t-1} + k_1\left(s_{i,t} - \rho\,\theta_{i,t-1}\right),$$

where k_1 is the stationary Kalman gain. k_1 is the solution to the system of two equations in two unknowns, k_1 and σ^2,

$$k_1 = \frac{\sigma^2}{\sigma^2 + \sigma_\eta^2}\;,\; \sigma^2 = \rho^2(1-k_1)\sigma^2 + \sigma_u^2.$$

The average stationary expectation then evolves according to

$$\bar{\theta}_t = (1-k_1)\rho\,\bar{\theta}_{t-1} + k_1\rho\,\theta_{t-1} + k_1\,u_t.$$

We can write the dynamics for $(\theta_t, \bar{\theta}_t)^\mathsf{T}$ in state space form, with transition equation

$$\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ k_1\rho & (1-k_1)\rho \end{bmatrix} \begin{pmatrix} \theta_{t-1} \\ \bar{\theta}_{t-1} \end{pmatrix} + \begin{bmatrix} 1 \\ k_1 \end{bmatrix} u_t$$

and measurement equation

$$z_{i,t} = (1\ ,\ 0) \begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} + \eta_{i,t}.$$

The stationary covariance matrix for the state given the history of signals up to t, $Var((\theta_t, \bar{\theta}_t)^\mathsf{T} | \{s_{i,t-j}\}_{j=0}^{\infty})$ can now be derived with a standard Kalman filter.

Table 3: Model parameter estimates $\phi = \{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$

	60	80	90	95	100	105	110	120	150	200
6	0.950	0.911	0.930	0.923	0.920	0.945	0.949	0.956	0.950	·
	(0.019)	(0.031)	(0.028)	(0.026)	(0.028)	(0.030)	(0.022)	(0.015)	(0.021)	·
12	0.967	0.930	0.939	0.949	0.941	0.930	0.949	0.967	0.969	·
	(0.015)	(0.024)	(0.027)	(0.028)	(0.022)	(0.025)	(0.022)	(0.013)	(0.017)	·
24	0.935	0.940	0.943	0.956	0.947	0.938	0.945	0.962	0.970	0.971
	(0.025)	(0.021)	(0.020)	(0.026)	(0.028)	(0.024)	(0.023)	(0.020)	(0.015)	(0.017)
36	0.939	0.943	0.941	0.969	0.952	0.932	0.958	0.948	0.963	0.946
	(0.022)	(0.026)	(0.023)	(0.026)	(0.023)	(0.021)	(0.025)	(0.021)	(0.017)	(0.021)
48	0.935	0.949	0.947	0.938	0.944	0.942	0.953	0.945	0.959	0.943
	(0.022)	(0.025)	(0.022)	(0.021)	(0.024)	(0.022)	(0.026)	(0.020)	(0.017)	(0.022)
60	0.982	0.956	0.951	0.941	0.948	0.941	0.945	0.956	0.963	0.937
	(0.012)	(0.028)	(0.022)	(0.021)	(0.021)	(0.025)	(0.024)	(0.027)	(0.016)	(0.026)
84	0.968	0.947	0.949	0.947	0.939	0.945	0.940	0.945	0.941	0.954
	(0.011)	(0.025)	(0.026)	(0.023)	(0.019)	(0.019)	(0.022)	(0.024)	(0.023)	(0.017)

(a) Mean and standard deviation ρ

	60	80	90	95	100	105	110	120	150	200
6	0.079	0.092	0.099	0.106	0.116	0.120	0.115	0.111	0.166	·
	(0.001)	(0.005)	(0.005)	(0.006)	(0.006)	(0.007)	(0.006)	(0.002)	(0.005)	·
12	0.047	0.076	0.082	0.086	0.091	0.095	0.095	0.073	0.135	·
	(0.001)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.002)	(0.003)	·
24	0.064	0.065	0.070	0.072	0.075	0.078	0.080	0.073	0.078	0.135
	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.005)
36	0.056	0.059	0.063	0.065	0.067	0.069	0.070	0.068	0.061	0.110
	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.003)
48	0.054	0.056	0.059	0.061	0.062	0.064	0.065	0.065	0.057	0.097
	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.002)	(0.002)
60	0.033	0.055	0.055	0.057	0.058	0.059	0.060	0.061	0.051	0.089
	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)
84	0.033	0.050	0.051	0.052	0.053	0.054	0.055	0.056	0.062	0.058
	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.001)

(b) Mean and standard deviation σ_u

	60	80	90	95	100	105	110	120	150	200
6	0.121	0.004	0.007	0.009	0.011	0.016	0.021	0.151	0.328	·
	(0.008)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.002)	(0.010)	(0.013)	·
12	0.055	0.004	0.006	0.007	0.009	0.011	0.014	0.036	0.262	·
	(0.004)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)	(0.016)	·
24	0.002	0.004	0.006	0.007	0.008	0.009	0.010	0.015	0.114	0.270
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.008)	(0.016)
36	0.003	0.004	0.005	0.006	0.007	0.008	0.008	0.011	0.054	0.183
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)	(0.012)
48	0.003	0.004	0.005	0.006	0.006	0.007	0.007	0.009	0.034	0.121
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)	(0.009)
60	0.032	0.002	0.004	0.005	0.005	0.006	0.006	0.007	0.024	0.083
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.006)
84	0.034	0.003	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.069
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)

(c) Mean and standard deviation σ_ε

	60	80	90	95	100	105	110	120	150	200
6	0.096	0.022	0.015	0.013	0.013	0.018	0.030	0.131	0.380	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.020)	·
12	0.041	0.014	0.010	0.010	0.010	0.011	0.015	0.036	0.281	·
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)	·
24	0.024	0.012	0.009	0.008	0.008	0.009	0.013	0.018	0.093	0.395
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.023)
36	0.021	0.011	0.009	0.008	0.008	0.009	0.010	0.015	0.049	0.231
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)
48	0.022	0.012	0.010	0.009	0.009	0.009	0.011	0.014	0.035	0.155
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.006)
60	0.025	0.012	0.010	0.009	0.009	0.010	0.011	0.014	0.027	0.116
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)
84	0.029	0.014	0.011	0.011	0.011	0.011	0.012	0.014	0.023	0.069
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)

(d) Mean and standard deviation σ_η

The panels in this table present the means and standard deviations (in parentheses) of the model parameter estimates. Panel (a) displays estimates of the persistence ρ of the AR1 process for the fundamental value. Panel (b) displays estimates of the standard deviation σ_u of the shock to the fundamental. Panel (c) displays the estimates of the standard deviation σ_ε of the noise in the consensus price. Panel (d) displays the estimates of the standard deviation of the noise σ_η in private signal. Estimates are obtained using MCMC methods assuming diffuse priors for all parameters. The first row and first column of each panel give moneyness and time-to-expiration, respectively, of the option contracts under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is December 2002 to February 2015.

Table 4: Kalman gains for $\theta_{i,t}$ and $\bar{\theta}_{i,t}$

	60	80	90	95	100	105	110	120	150	200
6	0.522	0.945	0.977	0.984	0.988	0.978	0.937	0.531	0.309	·
	(0.012)	(0.006)	(0.003)	(0.002)	(0.001)	(0.002)	(0.007)	(0.011)	(0.015)	·
12	0.649	0.966	0.984	0.987	0.989	0.986	0.976	0.825	0.344	·
	(0.010)	(0.004)	(0.002)	(0.001)	(0.001)	(0.002)	(0.003)	(0.010)	(0.013)	·
24	0.875	0.967	0.983	0.987	0.988	0.986	0.975	0.945	0.531	0.248
	(0.013)	(0.004)	(0.002)	(0.002)	(0.001)	(0.002)	(0.003)	(0.006)	(0.011)	(0.014)
36	0.878	0.966	0.981	0.984	0.986	0.983	0.978	0.955	0.672	0.330
	(0.012)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.005)	(0.011)	(0.015)
48	0.860	0.957	0.974	0.979	0.981	0.978	0.973	0.954	0.751	0.414
	(0.014)	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.005)	(0.012)	(0.015)
60	0.689	0.955	0.967	0.973	0.975	0.974	0.968	0.950	0.797	0.480
	(0.010)	(0.005)	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)	(0.006)	(0.012)	(0.016)
84	0.640	0.930	0.953	0.958	0.959	0.958	0.954	0.939	0.882	0.529
	(0.010)	(0.008)	(0.005)	(0.005)	(0.005)	(0.011)	(0.005)	(0.007)	(0.012)	(0.013)

(a) k_s

	60	80	90	95	100	105	110	120	150	200
6	0.081	0.051	0.018	0.010	0.006	0.012	0.040	0.096	0.116	·
	(0.010)	(0.006)	(0.002)	(0.001)	(0.001)	(0.001)	(0.005)	(0.011)	(0.009)	·
12	0.071	0.030	0.012	0.008	0.005	0.007	0.012	0.072	0.114	·
	(0.009)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.008)	(0.013)	·
24	0.132	0.028	0.012	0.008	0.006	0.007	0.014	0.030	0.089	0.152
	(0.015)	(0.003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.003)	(0.011)	(0.017)
36	0.126	0.029	0.013	0.010	0.008	0.009	0.013	0.028	0.096	0.139
	(0.015)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.003)	(0.011)	(0.016)
48	0.147	0.037	0.019	0.014	0.012	0.014	0.017	0.031	0.102	0.160
	(0.017)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.013)	(0.020)
60	0.079	0.045	0.026	0.020	0.018	0.019	0.023	0.038	0.099	0.190
	(0.010)	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.005)	(0.013)	(0.022)
84	0.093	0.067	0.042	0.037	0.035	0.036	0.039	0.054	0.117	0.117
	(0.012)	(0.008)	(0.005)	(0.004)	(0.004)	(0.013)	(0.005)	(0.006)	(0.014)	(0.015)

(b) k_p

	60	80	90	95	100	105	110	120	150	200
6	0.318	0.893	0.955	0.969	0.976	0.957	0.878	0.324	0.146	·
	(0.013)	(0.012)	(0.005)	(0.003)	(0.003)	(0.005)	(0.012)	(0.012)	(0.012)	·
12	0.454	0.933	0.968	0.975	0.978	0.971	0.953	0.688	0.172	·
	(0.013)	(0.007)	(0.004)	(0.003)	(0.002)	(0.003)	(0.005)	(0.016)	(0.011)	·
24	0.766	0.936	0.966	0.973	0.977	0.972	0.951	0.894	0.327	0.110
	(0.022)	(0.007)	(0.004)	(0.003)	(0.003)	(0.003)	(0.005)	(0.011)	(0.012)	(0.009)
36	0.771	0.932	0.962	0.969	0.972	0.967	0.957	0.912	0.475	0.157
	(0.022)	(0.007)	(0.004)	(0.003)	(0.003)	(0.004)	(0.005)	(0.009)	(0.015)	(0.012)
48	0.740	0.916	0.949	0.958	0.961	0.957	0.948	0.910	0.575	0.213
	(0.024)	(0.009)	(0.006)	(0.005)	(0.004)	(0.005)	(0.006)	(0.010)	(0.018)	(0.014)
60	0.501	0.911	0.936	0.948	0.951	0.948	0.938	0.903	0.642	0.262
	(0.013)	(0.010)	(0.007)	(0.006)	(0.005)	(0.006)	(0.007)	(0.011)	(0.019)	(0.016)
84	0.439	0.865	0.909	0.918	0.920	0.918	0.911	0.882	0.779	0.318
	(0.013)	(0.015)	(0.010)	(0.009)	(0.009)	(0.019)	(0.010)	(0.013)	(0.022)	(0.015)

(c) $\bar{k}_s$

	60	80	90	95	100	105	110	120	150	200
6	0.196	0.101	0.038	0.023	0.015	0.028	0.091	0.227	0.271	·
	(0.022)	(0.012)	(0.004)	(0.003)	(0.002)	(0.003)	(0.010)	(0.024)	(0.018)	·
12	0.173	0.062	0.026	0.018	0.013	0.017	0.029	0.171	0.266	·
	(0.020)	(0.007)	(0.003)	(0.002)	(0.002)	(0.002)	(0.004)	(0.017)	(0.027)	·
24	0.248	0.059	0.026	0.019	0.015	0.017	0.033	0.072	0.212	0.335
	(0.027)	(0.007)	(0.003)	(0.002)	(0.002)	(0.002)	(0.004)	(0.008)	(0.024)	(0.030)
36	0.240	0.061	0.030	0.023	0.019	0.022	0.030	0.065	0.226	0.318
	(0.026)	(0.007)	(0.004)	(0.003)	(0.002)	(0.003)	(0.004)	(0.007)	(0.024)	(0.032)
48	0.277	0.077	0.043	0.032	0.028	0.031	0.039	0.070	0.234	0.359
	(0.030)	(0.009)	(0.005)	(0.004)	(0.003)	(0.004)	(0.005)	(0.008)	(0.025)	(0.037)
60	0.190	0.088	0.056	0.044	0.040	0.041	0.051	0.083	0.226	0.412
	(0.022)	(0.011)	(0.007)	(0.005)	(0.005)	(0.005)	(0.006)	(0.010)	(0.024)	(0.039)
84	0.220	0.133	0.086	0.076	0.072	0.075	0.081	0.110	0.227	0.272
	(0.025)	(0.016)	(0.010)	(0.009)	(0.008)	(0.023)	(0.010)	(0.013)	(0.025)	(0.031)

(d) $\bar{k}_p$

This table presents the means and standard deviations (in parantheses) of the weight dealers put on "news" in the private signal and the consensus price when updating expectations about the fundamental value and the average expectation across dealers. The top panels display the Kalman gain for private signal (k_s; Panel (a)) and the consensus price (k_p; Panel (b)) w.r.t. the fundamental value as given in (10). The bottom panels display the Kalman gains for private signal ($\bar{k}_s$; Panel (c)) and consensus price ($\bar{k}_p$; Panel (d)) w.r.t. the average expectation, as given in (11). The first row and first column of each table give moneyness and time-to-expiration, respectively, of the options under consideration. The sample period of the data is December 2002 to February 2015.

Table 5: Weight k on new information and belief correlation ρ_{12}

	60	80	90	95	100	105	110	120	150	200
6	0.625	0.998	0.996	0.995	0.995	0.990	0.978	0.649	0.465	·
	(0.012)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.013)	(0.012)	·
12	0.733	0.998	0.996	0.996	0.995	0.993	0.989	0.901	0.490	·
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.010)	(0.014)	·
24	0.999	0.997	0.995	0.995	0.995	0.993	0.990	0.976	0.639	0.440
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.013)	(0.017)
36	0.997	0.995	0.995	0.995	0.994	0.993	0.991	0.984	0.781	0.513
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.012)	(0.017)
48	0.996	0.995	0.994	0.994	0.993	0.993	0.991	0.986	0.860	0.613
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.013)	(0.019)
60	0.778	0.999	0.994	0.994	0.994	0.993	0.992	0.989	0.900	0.703
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.015)	(0.018)
84	0.747	0.997	0.996	0.996	0.995	0.995	0.994	0.993	0.994	0.671
	(0.012)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.016)

(a) k

	60	80	90	95	100	105	110	120	150	200
6	0.954	1.000	1.000	1.000	1.000	1.000	1.000	0.958	0.915	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.003)	·
12	0.974	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.922	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)	·
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.956	0.909
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.004)
36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	0.925
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.004)
48	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.947
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
60	0.981	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.964
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
84	0.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.962
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)

(b) ρ_{12}

This table presents means and standard deviations (in parantheses) of k, the Kalman gain as given in (12) and ρ_{12}, the correlation between θ_t and $\bar{\theta}_t$ according to a dealer's posterior beliefs. $1-k$ measures how much weight a dealer puts on their prior when updating expectations about the fundamental value θ_t. The first row and first column of each table give moneyness and time-to-expiration, respectively, of the options under consideration. The sample period of the data is December 2002 to February 2015.

Table 6: 95% posterior intervals θ_t and $\bar{\theta}_t$

	60	80	90	95	100	105	110	120	150	200
6	9.677	2.274	1.300	1.045	0.903	1.133	1.690	5.065	12.845	·
	(0.161)	(0.026)	(0.014)	(0.011)	(0.010)	(0.012)	(0.019)	(0.078)	(0.391)	·
12	4.052	1.369	0.896	0.772	0.706	0.786	0.943	1.858	8.679	·
	(0.057)	(0.015)	(0.010)	(0.008)	(0.007)	(0.008)	(0.010)	(0.024)	(0.202)	·
24	2.586	1.122	0.781	0.685	0.640	0.681	0.888	1.117	3.767	11.038
	(0.037)	(0.012)	(0.008)	(0.007)	(0.007)	(0.007)	(0.010)	(0.013)	(0.060)	(0.389)
36	2.181	1.042	0.763	0.689	0.655	0.695	0.776	1.024	2.396	7.533
	(0.031)	(0.012)	(0.008)	(0.008)	(0.007)	(0.008)	(0.009)	(0.012)	(0.036)	(0.216)
48	2.249	1.113	0.852	0.771	0.734	0.764	0.833	1.037	1.981	5.876
	(0.034)	(0.013)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)	(0.012)	(0.029)	(0.144)
60	2.320	1.144	0.921	0.832	0.799	0.816	0.885	1.068	1.680	5.018
	(0.033)	(0.013)	(0.011)	(0.009)	(0.009)	(0.009)	(0.010)	(0.013)	(0.024)	(0.119)
84	2.623	1.343	1.077	1.017	0.993	0.994	1.037	1.182	1.645	3.480
	(0.040)	(0.018)	(0.014)	(0.013)	(0.012)	(0.019)	(0.013)	(0.015)	(0.025)	(0.071)

(a) $3.92 \cdot \sigma^p_{11}$

	60	80	90	95	100	105	110	120	150	200
6	6.287	2.148	1.270	1.029	0.892	1.108	1.584	3.282	6.522	·
	(0.087)	(0.022)	(0.013)	(0.011)	(0.009)	(0.011)	(0.017)	(0.043)	(0.143)	·
12	2.938	1.323	0.882	0.762	0.698	0.775	0.921	1.557	4.725	·
	(0.034)	(0.014)	(0.009)	(0.008)	(0.007)	(0.008)	(0.010)	(0.017)	(0.093)	·
24	2.262	1.086	0.768	0.676	0.632	0.671	0.867	1.056	2.473	5.114
	(0.027)	(0.011)	(0.008)	(0.007)	(0.007)	(0.007)	(0.009)	(0.011)	(0.034)	(0.131)
36	1.914	1.006	0.749	0.679	0.646	0.683	0.759	0.978	1.743	3.857
	(0.023)	(0.011)	(0.008)	(0.007)	(0.007)	(0.007)	(0.008)	(0.010)	(0.021)	(0.090)
48	1.934	1.065	0.830	0.755	0.720	0.748	0.811	0.989	1.537	3.240
	(0.025)	(0.011)	(0.009)	(0.008)	(0.008)	(0.008)	(0.009)	(0.011)	(0.019)	(0.066)
60	1.733	1.093	0.891	0.810	0.779	0.795	0.857	1.014	1.361	2.894
	(0.020)	(0.012)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)	(0.011)	(0.017)	(0.057)
84	1.866	1.249	1.026	0.974	0.952	0.952	0.990	1.110	1.451	2.219
	(0.024)	(0.014)	(0.012)	(0.011)	(0.011)	(0.023)	(0.011)	(0.013)	(0.018)	(0.036)

(b) $3.92 \cdot \sigma^p_{22}$

This table presents the means and standards deviations (in parentheses) of the 95% posterior intervals, as implied by a dealer's posterior beliefs, for the fundamental value, $[3.92 \cdot \sigma^p_{11}]$, as given in (9) and for the average expectation, $[3.92 \cdot \sigma^p_{22}]$. The lengths of the posterior intervals are given in terms of volatility points. The first row and first column of each table give moneyness and time-to-expiration, respectively, of the options under consideration. The sample period of the data is December 2002 to February 2015.

Table 7: Counterfactual experiments - change in uncertainty

	60	80	90	95	100	105	110	120	150	200
6	1.38	0.12	0.02	0.01	0.00*	0.01	0.11	1.58	2.91	·
	(0.17)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.18)	(0.29)	·
12	0.93	0.05	0.01	0.00*	0.00*	0.00*	0.01	0.50	2.85	·
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.32)	·
24	0.62	0.04	0.01	0.01	0.00*	0.00*	0.02	0.07	1.52	4.36
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.18)	(0.50)
36	0.59	0.04	0.01	0.01	0.01	0.01	0.01	0.06	1.14	3.20
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.14)	(0.38)
48	0.76	0.07	0.02	0.01	0.01	0.01	0.02	0.06	0.92	3.06
	(0.15)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.12)	(0.36)
60	0.93	0.09	0.04	0.02	0.02	0.02	0.03	0.09	0.76	3.06
	(0.12)	(0.02)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.02)	(0.10)	(0.34)
84	1.21	0.20	0.09	0.07	0.06	0.07	0.08	0.14	0.53	1.90
	(0.15)	(0.04)	(0.02)	(0.02)	(0.01)	(0.10)	(0.02)	(0.03)	(0.11)	(0.24)

(a) *Reduction in valuation uncertainty*: Δ_1^p

	60	80	90	95	100	105	110	120	150	200
6	9.65	0.70	0.12	0.05	0.03	0.09	0.77	10.82	19.17	·
	(1.02)	(0.16)	(0.03)	(0.01)	(0.01)	(0.02)	(0.16)	(1.08)	(1.40)	·
12	6.70	0.28	0.06	0.03	0.02	0.04	0.10	3.63	18.27	·
	(0.75)	(0.06)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.42)	(1.66)	·
24	3.48	0.26	0.06	0.04	0.03	0.03	0.12	0.54	10.37	27.33
	(0.66)	(0.06)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.11)	(1.09)	(2.33)
36	3.33	0.28	0.08	0.05	0.04	0.05	0.09	0.40	7.98	20.69
	(0.63)	(0.06)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.09)	(0.83)	(1.93)
48	4.25	0.44	0.15	0.10	0.08	0.10	0.15	0.44	6.50	19.34
	(0.79)	(0.10)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.10)	(0.73)	(1.84)
60	6.63	0.53	0.25	0.16	0.14	0.15	0.22	0.56	5.29	19.08
	(0.75)	(0.12)	(0.06)	(0.04)	(0.03)	(0.03)	(0.05)	(0.12)	(0.61)	(1.70)
84	8.45	1.18	0.53	0.43	0.39	0.44	0.49	0.87	3.08	12.67
	(0.93)	(0.26)	(0.12)	(0.10)	(0.09)	(0.52)	(0.11)	(0.19)	(0.60)	(1.35)

(b) *Reduction in strategic uncertainty*: Δ_2^p

	60	80	90	95	100	105	110	120	150	200
6	12.02	0.01	0.00*	0.00*	0.00*	0.01	0.07	11.35	27.01	·
	(0.52)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.52)	(1.03)	·
12	6.15	0.00*	0.00*	0.00*	0.00*	0.00*	0.01	1.02	25.45	·
	(0.35)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.16)	(0.84)	·
24	0.01	0.01	0.00*	0.00*	0.00*	0.00*	0.01	0.07	11.99	33.46
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.52)	(1.21)
36	0.02	0.01	0.00*	0.00*	0.00*	0.01	0.01	0.03	4.71	24.14
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.34)	(1.00)
48	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.03	2.19	16.80
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.27)	(0.82)
60	4.60	0.00*	0.01	0.01	0.01	0.01	0.01	0.03	1.24	11.55
	(0.33)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.24)	(0.74)
84	6.13	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.04	10.88
	(0.39)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.63)

(c) *Reduction in valuation uncertainty*: Δ_1^θ

	60	80	90	95	100	105	110	120	150	200
6	41.31	0.02	0.01	0.01	0.01	0.04	0.27	38.80	62.69	·
	(1.43)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.07)	(1.49)	(1.09)	·
12	24.49	0.01	0.01	0.01	0.01	0.02	0.05	4.35	61.23	·
	(1.28)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.74)	(1.52)	·
24	0.03	0.02	0.01	0.01	0.01	0.02	0.05	0.27	40.68	62.10
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.07)	(1.48)	(1.95)
36	0.07	0.03	0.02	0.02	0.02	0.02	0.04	0.13	18.68	57.87
	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.04)	(1.31)	(1.75)
48	0.09	0.03	0.03	0.02	0.02	0.03	0.04	0.12	8.94	46.67
	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(1.16)	(1.89)
60	18.72	0.01	0.03	0.02	0.02	0.03	0.04	0.10	5.06	35.14
	(1.29)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(1.05)	(1.94)
84	23.65	0.04	0.03	0.03	0.03	0.03	0.04	0.07	0.13	36.56
	(1.41)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.04)	(1.82)

(d) *Reduction in strategic uncertainty*: Δ_2^θ

The panels in this table present the counterfactual percentage decreases in valuation and strategic uncertainty. The two top panels display the reductions in uncertainties when comparing a setting without consensus price to a setting with consensus price. Panel (a) presents the results for the percentage decrease in **valuation uncertainty**, Δ_1^p in (13). Panel (b) shows the percentage increase in **strategic uncertainty**, Δ_2^p. The lower panels shows the counterfactual percentage reductions in valuation and strategic uncertainty when comparing the current information structure to an information structure with a consensus price that perfectly reveals last period's state. Panel (c) shows percentage reduction in **valuation uncertainty**, Δ_1^θ in (13). Panel (d) shows the percentage reduction in **strategic uncertainty**, Δ_2^θ. The first row and first column of each table give moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviations of the posterior distribution of the parameter is given in parentheses below the means (0.00 signifies standard deviations below 0.005). The sample period is from December 2002 to February 2015.

8 Internet Appendix

8.1 The value of information in OTC markets

This is a simple one-period model to illustrate the value of the consensus price information for dealers in the OTC options market. It shows that dealers that use an interdealer market to share risk are naturally concerned about both fundamental asset values and other dealers' valuation. A dealer is willing to pay for information that reduces its uncertainty in any of these two dimensions.

The model

Before entering the market, every dealer $i \in [0,1]$ observes a private signal about the fundamental value of an option, given by the random variable θ. She can also pay to receive a public signal about that value. For now, the exact form of these signals is not important. The game proceeds in three steps:

1. Dealer $i \in [0,1]$ decides whether to buy the public signal at cost f.

2. After observing signal(s), the dealer enters the market and is matched with a client. A client is a buyer or seller of one option contract with equal probability. The dealer can credibly communicate her valuation of the option to the client. The client is willing to pay (receive) at most Δ in excess of (below) the dealer's valuation.

3. After buying or selling the option from the client, dealer i enters the interdealer market. She is matched with a dealer with opposite option inventory with probability $0 \leq \gamma \leq 1$. If matched, dealers trade at the average expectation of fundamental values among active dealers denoted by $\bar{\theta}$.[28]

4. If a dealer has not been matched in the interdealer market (probability $1 - \gamma$) she hedges the option herself. At expiry, she receives the fundamental value θ but hedging physically creates a cost of $c > 0$.

[28]We do not explicitly model the trading mechanism that would yield this as a market-clearing price of interdealer market.

Pricing after entry

Suppose dealer i is matched with a client that wants to buy. The dealer charges a price a_i to the client. If the dealer is matched in the interdealer market, her profit is $a_i - \bar{\theta}$. Otherwise her profit is $a_i - \theta - c$. We assume that the dealer minimizes a loss function that is quadratic in losses.[29] The pricing problem is then

$$\mathcal{L}_i^s = \min_a \mathbb{E}_i \left\{ \gamma(a - \bar{\theta} - \pi)^2 + (1-\gamma)(a - \theta - c - \pi)^2 \right\},$$

where the expectation is taken over dealer i's information set when she is interacting with the client, that is after entry and having observed signals, but before entering the interdealer market. The first-order condition for a yields the optimal price,

$$a_i^* = \pi + \gamma \, \mathbb{E}_i \, \bar{\theta} + (1-\gamma)\mathbb{E}_i \, (\theta + c).$$

We assume that dealer i can credibly communicate the "fair value" of the option, namely $\gamma \, \mathbb{E}_i \, \bar{\theta} + (1-\gamma)\mathbb{E}_i \, (\theta + c)$, to her client. For the client to buy, we further assume that the markup in the optimal price is smaller than the client's maximal willingness to pay, that is $\pi \leq \Delta$.

Substituting a_i^* back into the loss function we find

$$\mathcal{L}_i^s = \gamma \, \mathbb{E}_i \left(\bar{\theta} - \bar{\theta}_i\right)^2 + (1-\gamma) \, \mathbb{E}_i \left(\theta - \theta_i\right)^2 + \gamma(1-\gamma)(\delta_i + c)^2,$$

where $\delta_i = \theta_i - \bar{\theta}_i$.

The case for a dealer buying from a client at price b is symmetric with loss function

$$\mathcal{L}_i^b = \min_b \mathbb{E}_i \left\{ \gamma(\bar{\theta} - b - \pi)^2 + (1-\gamma)(\theta - b - c - \pi)^2 \right\}.$$

It yields a nearly identical loss function to the case of buying from a client, namely,

$$\mathcal{L}_i^b = \gamma \, \mathbb{E}_i \left(\bar{\theta} - \bar{\theta}_i\right)^2 + (1-\gamma) \, \mathbb{E}_i \left(\theta - \theta_i\right)^2 + \gamma(1-\gamma)(\delta_i - c)^2.$$

[29] This captures the idea that dealers' institutions prefer smooth profits with target level π.

Participation decision

The ex-ante expected loss of dealer i with signals $\mathbf{s}_i$ is

$$-\mathbb{E}\left(\frac{1}{2}\mathcal{L}_i^s + \frac{1}{2}\mathcal{L}_i^b \mid \mathbf{s}_i\right) = -\gamma\,\mathrm{Var}(\bar{\theta} \mid \mathbf{s}_i) - (1-\gamma)\mathrm{Var}(\theta \mid \mathbf{s}_i) \\ - \gamma(1-\gamma)\,\mathbb{E}(\,\delta_i^2 \mid \mathbf{s}_i) - \gamma(1-\gamma)\,c^2.$$

The dealer buys the public signal if the reduction in expected loss exceeds the price of the signal, which is f.

The public signal is valued as it allows for better pricing decisions. Its ability to reduce strategic uncertainty is valued as it helps to predict prices in the interdealer market.

8.2 IHS Markit's Totem submission process

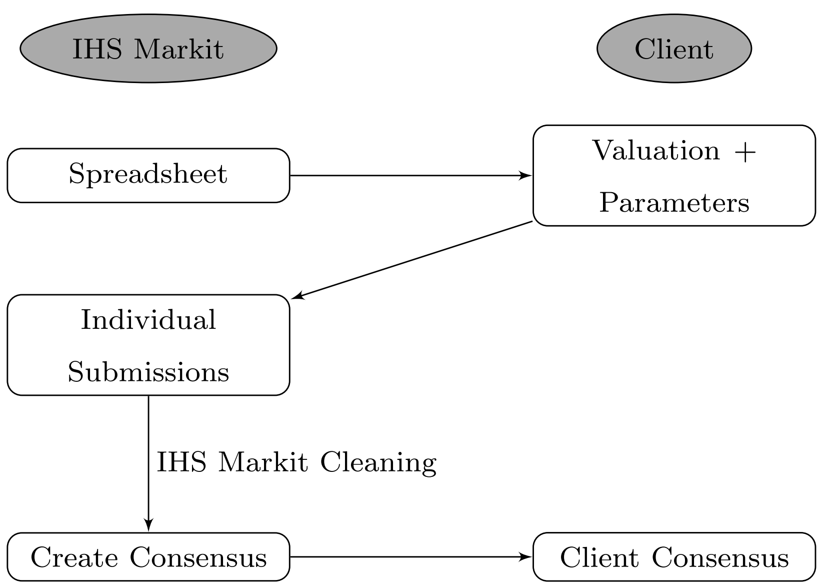

Figure 6: Diagram – Submission process

Figure 6 depicts a diagram of the submission process to IHS Markit's Totem service for plain vanilla index options.[30] On the last trading day of the month, Totem issues a spreadsheet to the n dealers that participate in the service. Dealers have to submit estimates for the mid price, defined as the average of bid price and offer price, of a range of put options with a moneyness between 80 and 100 and a range of call options with a moneyness ranging from 100 to 120 with a time-to-expiration of 6 months. Dealers that want to submit prices for different contracts are required to submit to all the available strike price and time-to-expiration combinations that lie in between the required contracts and the additionally demanded contracts.

We denote submitter i's estimate for the mid-price of an out-of-the-money (OTM) put with moneyness K, defined as the strike price of the option divided by the spot price of the underlying asset times 100, and time-to-expiration T (in days) by $P^i\left(p, K, T\right)$ and the mid-price estimate for an OTM call option with the same moneyness and time-to-expiration by $P^i\left(c, K, T\right)$. Submitter i also needs to submit the following input in addition to the mid-price estimate:

- the discount factor $\beta^i\left(T\right)$,
- the reference level $R^i(T)$, that is the price of a futures contract with maturity date closest to the valuation date,
- and the implied spot level $S^i\left(K, T\right)$, that is the implied level of the underlying index of the futures contract.

Submitters are provided with precise instructions for the timing of the valuation and the reference level that is to be used. To address any issues which might still arise with respect to valuation timing and the effect it could have on the comparability of prices across submitters, the submitted prices are aligned according to a predefined mechanism. The average consensus-implied spot from the at-the-money 6-month option, that is $\bar{S}\left(100, 6\right)$, is used for all other combinations of K and T. The submitted prices are restated in terms of $\bar{S}\left(K, T\right)$,

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giving: $\widehat{p}^i\left(\{c,p\},K,T\right) = {}^{P^i(\{c,p\},K,T)}/_{\bar{S}(K,T)}$.

Given the submitted quantities, a Totem analyst calculates various implied quantities to validate the individual submissions. Put-call parity for ATM options is used to retrieve the relative forward, i.e.,

$$f^i\left(K,T\right) = \frac{\widehat{p}^i\left(c,K,T\right) - \widehat{p}^i\left(p,K,T\right)}{\beta^i\left(T\right)} + 1$$

The above inputs are then used in the Black-Scholes model,

$$\widehat{p}^i\left(c,K,T\right) = \beta^i\left(T\right)\left[f^i\left(K,T\right)N\left(d_{1,i}\right) - KN\left(d_{2,i}\right)\right]$$

$$d_{1,i} = \frac{\ln\left(\frac{f^i}{K}\right) + \left(\frac{(\sigma^i)^2}{2}\right)T^a}{\sigma^i\sqrt{T^a}} \quad \text{, where} \quad T^a = \frac{\mathrm{T}}{365.25}$$

$$d_{2,i} = d_{1,i} - \sigma^i\sqrt{T^a}$$

to back-out σ^i in the above expression, which yields the implied volatility (IV) corresponding to submitter i's price submission for the given contract. We denote this IV by $\sigma_i(K,T)$. Here $N()$ is the cdf of the standard normal distribution.

When reviewing submissions, Totem analysts compare these IVs against other submitted prices and market conditions. They take the following points into consideration:

- the number of contributors,
- market activity & news,
- market conventions,
- the distribution and spread of contributed data,
- and, in a "one way market," they check if the concept of a mid-market price is clearly understood.

In addition to these criteria, analysts also visually inspect the ATM implied volatility term structure and the shape of the implied volatility curve for a given term, also referred to as the skew or the smile. After the vetting process, the analyst proceeds to the aggregation of the individual submissions into the consensus data.

Submitters' implied volatilities $\sigma_i(K,T)$ are aggregated into the consensus IV,

$$\bar{\sigma}\left(K,T\right)=\frac{1}{n(K,T)}\sum_{i=1}^{n(K,T)}\sigma_i\left(K,T\right).$$

Here $n(K,T)$ is the number of IVs used to calculate the consensus IV. Given more than 6 non-rejected IVs are available, the highest and lowest IV are excluded in the calculation to obtain a robust consensus IV. The same process takes place for the submitted prices to calculate a consensus price.

Submitters whose pricing information has been accepted by the Totem service receive the consensus information within 5 hours of the submission deadline. The consensus data include the average, standard deviation, skewness, and kurtosis of the distribution of accepted prices and implied volatilities. They also include the number of submitters to the consensus data.

8.3 Demand-based option pricing

Here, we show how the assumed AR(1) process for the fundamental value of the option (expressed in terms of the logarithm of its implied volatility), i.e.

$$\theta_t=\rho\,\theta_{t-1}+\sigma_u\,u_t \;\text{ with }\; u_t\sim N(0,1),$$

can be obtained within the framework of demand-based option pricing developed in Gârleanu et al. (2009). The framework shows how demand pressures can influence option prices when option dealers are risk averse and asset markets are not frictionless. We refer to the paper for details.

The price of the asset that underlies the options contract follows a geometric Brownian motion, i.e.

$$dS_t=\mu\,S_t\,dt+\sigma\,S_t\,dW_t.$$

In the absence of demand pressure, each option has a constant Black-Scholes implied volatility of σ, that is the volatility surface is flat in all periods.

We assume that the only source of friction is the inability to hedge options continuously. For this case, Gârleanu et al. (2009) show that the Black-Scholes IV for option i changes with a shift in demand for option j according to

$$\frac{\partial \sigma_t^i}{\partial d_t^j} = \frac{\gamma\, r\, Var_t\left((\Delta S)^2\right)}{4}\, \frac{f_{SS}^i}{\nu^i}\, f_{SS}^j + o\left(\Delta_t^2\right)$$

where f^i is the Black-Scholes (BS) price of option i, f_{SS}^i is option i's BS gamma and ν^i is option i's BS vega, r is the risk-free rate, and γ is the coefficient of relative risk aversion of the risk-averse dealers with CRRA utility. Δ_t is the time interval between two re-hedging opportunities, the only source of friction in this model.

As the price of the underlying, S, follows Brownian motion we have

$$\Delta S = S_{t+\Delta_t} - S_t \approx \mu\, S_t\, \Delta_t + \sigma \sqrt{\Delta_t}\, S_t\, \varepsilon,$$

where $\varepsilon \sim N(0,1)$. It follows that

$$Var_t\left((\Delta S)^2\right) = Var_t\left(\sigma^2\, \Delta_t\, S_t^2\, \varepsilon^2 + 2\mu\, \sigma\, S_t^2\, \Delta_t^{3/2}\, \varepsilon\right) = 2\sigma^4\, S_t^4\, \Delta_t^2 + o\left(\Delta_t^2\right),$$

BS gamma of option i is

$$f_{SS}^i = \frac{\phi(d_1)}{\sigma\, S_t\, \sqrt{\tau}},$$

and BS vega of option i is

$$\nu^i = S_t\, \sqrt{\tau}\, \phi(d_1),$$

where

$$d_1 = \frac{\log(S/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

Thus, using the above result from Gârleanu et al. (2009), the change in the IV of option i induced by a marginal change in demand for this option is

$$\frac{\partial \sigma_t^i}{\partial d_t^i} = \frac{1}{2}\, \gamma\, r \left(\frac{\sigma\, \Delta_t}{\sqrt{\tau}}\right)^2 \frac{\phi(d_1)}{\sqrt{\tau}}\, S_t + o\left(\Delta_t^2\right).$$

Here, d_t^i is client demand for option i in units of options. We define the corresponding dollar demand for option i as

$$\hat{d}_t^i = d_t\, p_t^i = \kappa^i\, S_t\, d_t,$$

where κ^i is a function of moneyness K/S, σ, and τ only. It follows that

$$\frac{\partial \sigma_t^i}{\partial \hat{d}_t^i} \approx \frac{1}{2}\,\gamma\, r \left(\frac{\sigma\,\Delta_t}{\sqrt{\tau}}\right)^2 \frac{\phi(d_1)}{\kappa^i\,\sqrt{\tau}}.$$

Also note that

$$\frac{\partial \log\, \sigma_t^i}{\partial \hat{d}_t^i} = \frac{\partial \sigma_t^i}{\partial \hat{d}_t^i}\frac{1}{\sigma_t^i},$$

which, for σ_t^i close to its long-run mean σ^i, implies that

$$\frac{\partial \log\, \sigma_t^i}{\partial \hat{d}_t^i} \approx \frac{1}{2}\,\gamma\, r \left(\frac{\sigma\,\Delta_t}{\sqrt{\tau}}\right)^2 \frac{\phi(d_1)}{\kappa^i\,\sigma^i\sqrt{\tau}} \equiv \lambda^i.$$

We assume that the impact of dollar demand for an option $j \neq i$ on the IV of option i is negligible. Let $\bar{d}^i$ denote the mean of $\hat{d}_t^i$. Then for σ_t^i close to σ^i and $\hat{d}_t^i$ close to $\bar{d}^i$ we approximately have

$$\log \sigma_t^i = \log \sigma^i + \lambda^i \left(\hat{d}_t^i - \bar{d}^i\right) = \left(\log \sigma^i - \lambda^i \bar{d}^i\right) + \lambda^i\, \hat{d}_t^i.$$

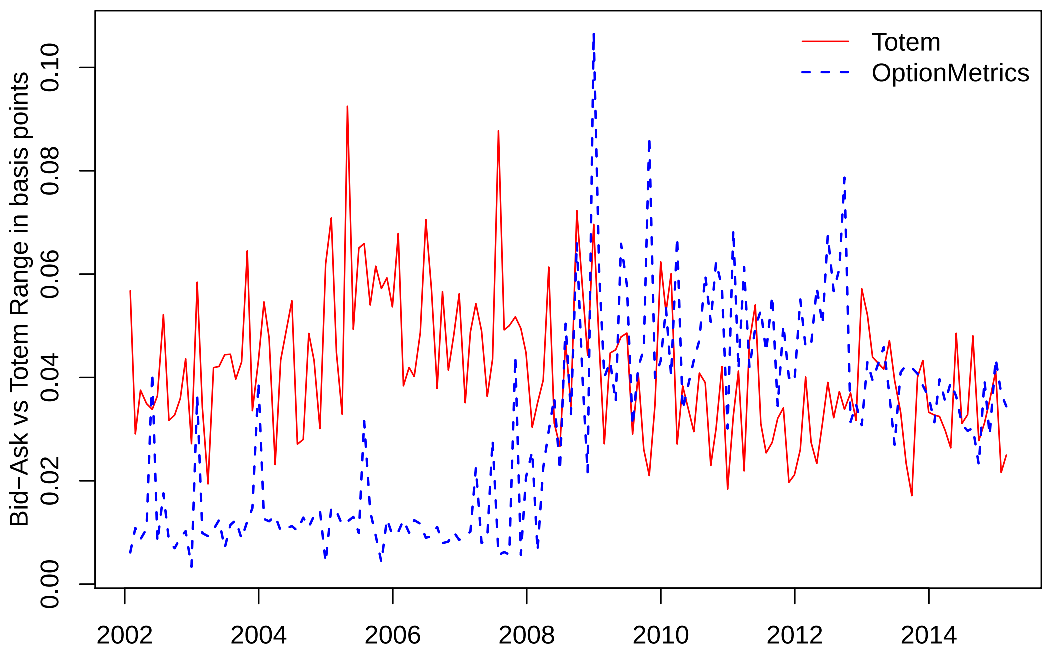
Now suppose dollar demand for option i follows an AR1 process,

$$\hat{d}_t^i = (1-\rho^i)\bar{d}^i + \rho^i\, \hat{d}_{t-1}^i + e_t^i.$$

Substituting this process into the previous expression for $\log \sigma_t^i$ yields an AR1 process for log IV that is driven by the demand shock e_t^i,

$$\log \sigma_t^i = (1-\rho^i)\log \sigma^i + \rho^i\, \log \sigma_{t-1}^i + \lambda^i\, e_t^i.$$

Figure 7: Bid-Ask spread vs Totem's submission range



The figure above displays the cross-sectional range (the difference between the highest and lowest submitted price) of the submissions to IHS Markit's Totem service and the bid-ask spread on traded options from OptionMetrics data. The series on display are for an option contract with time-to-expiration of 6 months and moneyness 100. The bid-ask spread is given by the difference between the best closing bid price and best closing ask price across all US option exchanges. On a given Totem valuation date, we match OptionMetrics option contracts that are a close proxy to a time-to-expiration of 6 months and moneyness 100. In the OptionMetrics database we search for contracts with a ± 10 days-to-maturity and a ±1 moneyness on Totem's valuation date. When multiple options match the criteria, an average of their bid-ask spread is taken.